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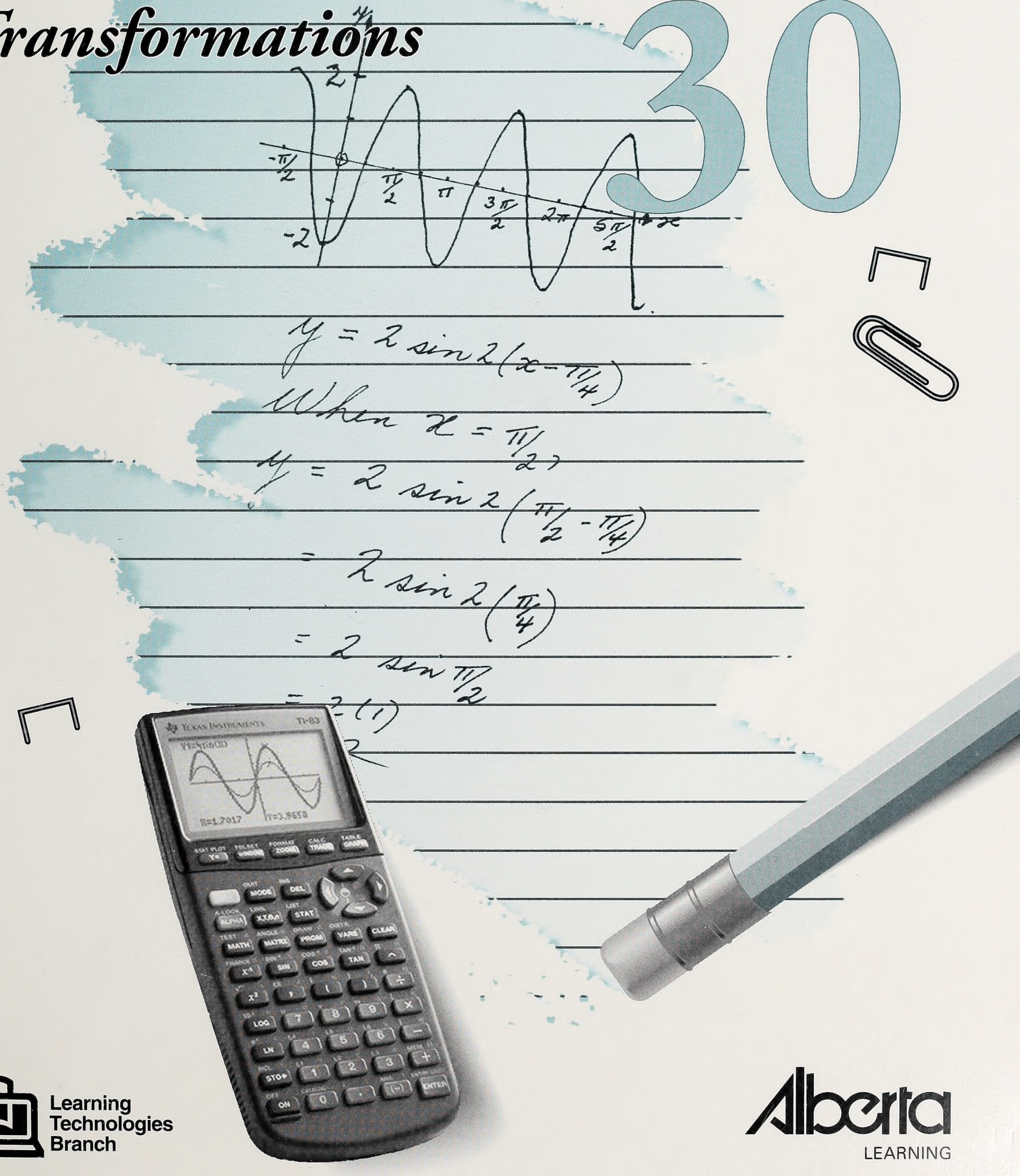
Module 1

MATHEMATICS 30

Transformations



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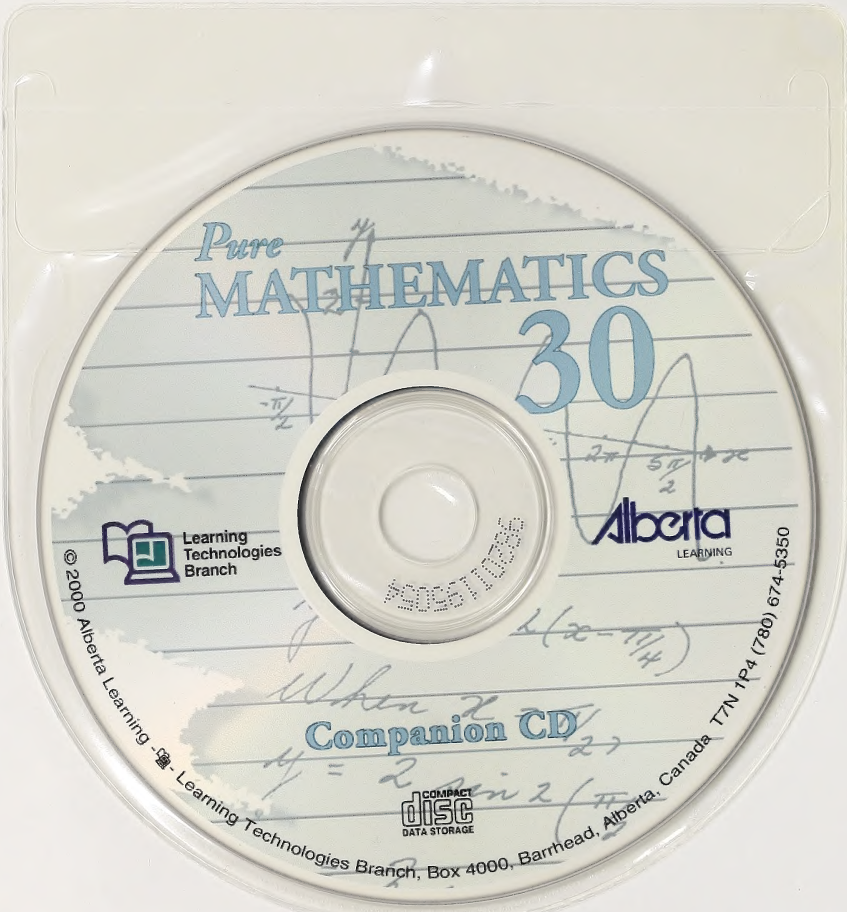
Pure
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30
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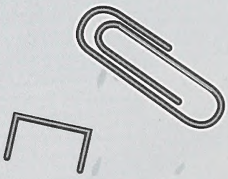
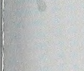
Pure MATHEMATICS 30

Module 1
Transformations





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Pure Mathematics 30
Module 1: Transformations
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-1631-5

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://www.learning.gov.ab.ca/ltb

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Welcome

Pure Mathematics 30

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Transformations

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Exponents and Logarithms

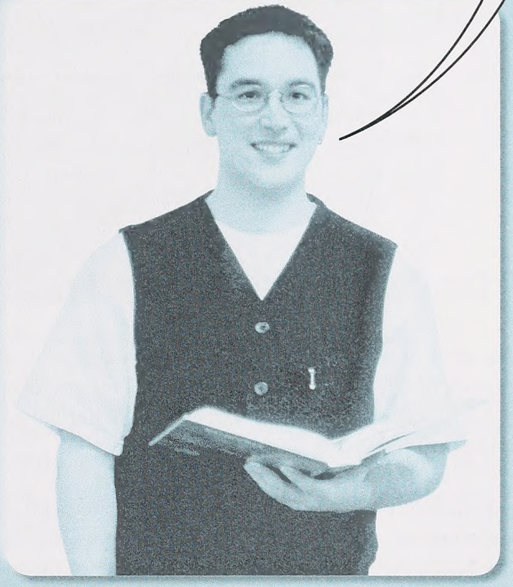
Module 3
Sequences and Series

Module 4
Conics

Module 5
Trigonometry

Module 6
Combinatorics

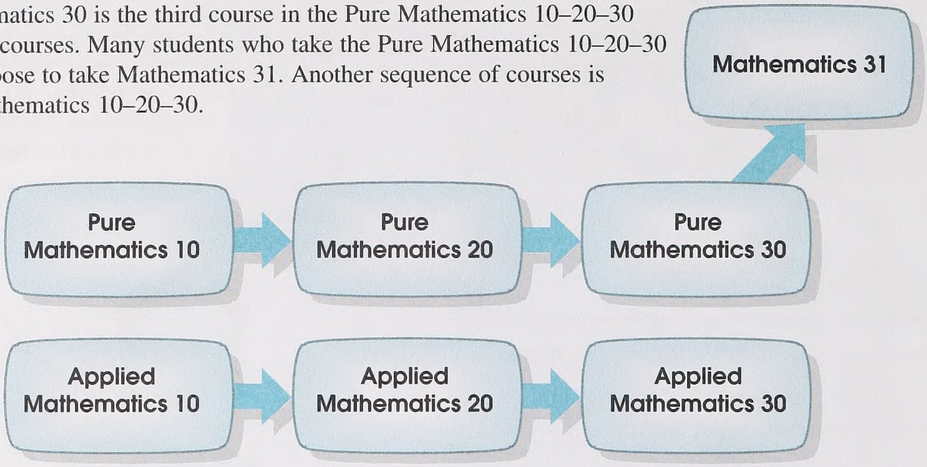
Module 7
Probability and Statistics


Welcome to **Module 1**. We hope you'll enjoy your study of **Transformations**.

Pure Mathematics 30 contains seven modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

High School Mathematics Courses

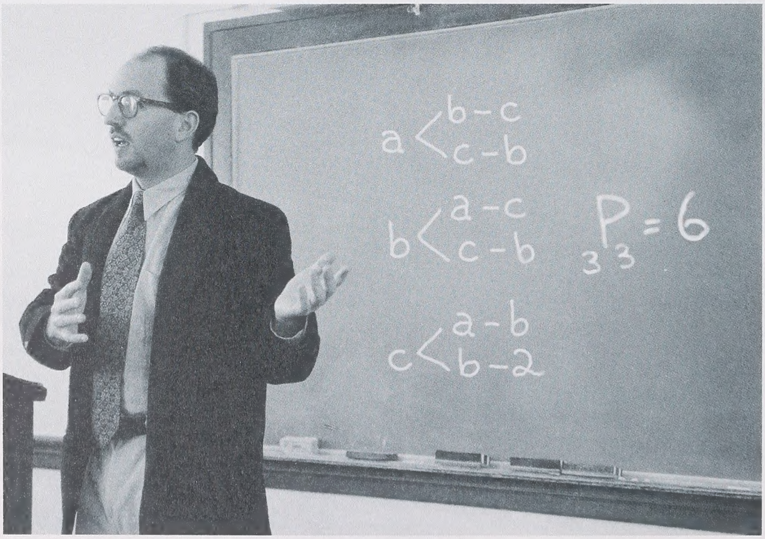
Pure Mathematics 30 is the third course in the Pure Mathematics 10–20–30 sequence of courses. Many students who take the Pure Mathematics 10–20–30 will also choose to take Mathematics 31. Another sequence of courses is Applied Mathematics 10–20–30.



How Do the Sequences Differ?

Each sequence of courses is designed for students with different mathematical strengths and interests. The Pure Mathematics 10–20–30 sequence is intended for students who are strong in mathematical theory and algebra. The Applied Mathematics 10–20–30 sequence is better suited to students who prefer to solve mathematical problems using numerical reasoning or geometry.

Each sequence of courses is designed for students with different career plans. Students who plan to take courses such as commerce, engineering, physics, and engineering technologies in post-secondary education will need to take Pure Mathematics 30 in high school. Some of these students will also need to take Mathematics 31.



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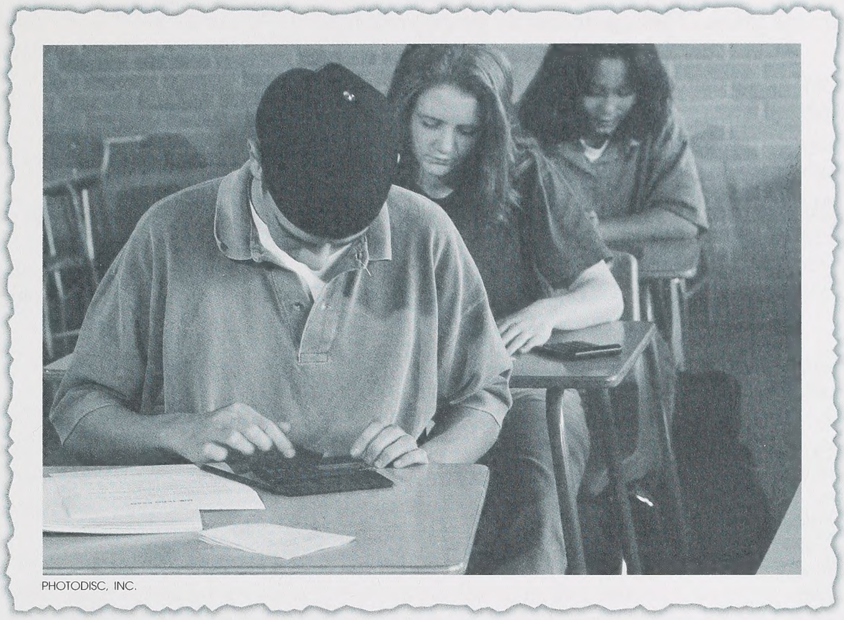
Where Can I Obtain Diploma Examination Information?



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Alberta Students will write a diploma examination at the end of the course. Alberta Learning provides several documents to help students prepare for this examination. These documents are found in the directory under "Students and Learning" on the Alberta Learning website, **http://www.learning.gov.ab.ca**. Information like course expectations, the makeup of the diploma examination, keyed copies of previous examinations, preparation guides, and calculator policies are available to students at this site.

Each year, in February and September, Alberta Learning provides teachers with information on a **student project**, which teachers **may** use as part of your overall assessment. Information to students will also be posted on the Alberta Learning website. Check with your teacher to determine what you will be expected to do. Be aware that one of the diploma examination's written-response questions is worth 10% of your diploma examination mark and will deal with elements of this project.

You should take advantage of the many sources of information about Pure Mathematics 30. Your success depends on your understanding of course expectations and evaluation procedures. Work closely with your teacher and do not hesitate to ask questions.

Remember, take the initiative to find out all you can about Pure Mathematics 30.

Can I Transfer to the Applied Mathematics Sequence of Courses?

If you do not successfully complete Pure Mathematics 30, you can repeat the course or you can transfer to the Applied Mathematics sequence. If you decide to transfer to the Applied Mathematics sequence, you may take the five-credit course called Applied Mathematics 20b or you may take the five-credit course called Applied Mathematics 20.

Mathematical Processes

Throughout the course you will be expected to do the following:

- communicate mathematically
- connect mathematical ideas to everyday experiences and to concepts in other disciplines
- develop and use estimation strategies
- develop and use mental math strategies
- develop and use problem-solving strategies
- reason and justify your thinking
- select and use appropriate technologies as tools to solve problems
- use visualization to assist in processing information, making connections, and solving problems

Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

This icon will show which mathematical process you are expected to perform.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Pure Mathematics 30 to be your study partner. You may find that having a friend to discuss mathematics with makes your studying more enjoyable.

You are also encouraged to record your mathematical ideas in a journal. Here are some suggestions for organizing your journal:

- Keep a section on new concepts and procedures you have learned. Get in the habit of describing new concepts, procedures, and strategies in your own words. If you are having difficulty, write your questions in your journal before discussing them with your teacher or study partner. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, or organizational charts) to help you connect mathematical ideas.

- Keep a section on your mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you finally grasped a difficult concept (an “aha!”) or when you first used a calculator or spreadsheet program in a new way.

- Keep a section on mathematics in the everyday world. This could include newspaper articles, cartoons, and descriptions of careers that require a mathematical background.

Resources You Will Need

In order to complete Pure Mathematics 30 successfully, you will need to have access to the following resources, which can be purchased through the Learning Resources Distributing Centre:

- the *MATHPOWER™ 12* textbook, Western Edition, published by McGraw-Hill Ryerson (2000)
- a graphing calculator

  Any of the following graphing calculators may be used: *Texas Instruments* TI-83, TI-83 Plus, TI-82, TI-86, TI-89, TI-92, or TI-92 Plus; *Casio* CFX-9850Ga-Plus, CFX-9850G, CFX-9800G, or FX-9700 series; or *Sharp* EL-9600c, EL-9600, EL-9300, or EL-9200. **Note:** Some of these calculators are no longer commercially manufactured.

- a computer and a spreadsheet program

  Any of the following spreadsheet programs may be used: *ClarisWorks™ 4.0* (or higher) for Macintosh, *ClarisWorks™ 4.0* (or higher) for Windows, *Microsoft® Works 3.0* (or higher) for Windows, or *Microsoft® Access 97* and *Microsoft® Excel 97* for Windows (a part of *Microsoft® Office Professional 95* or *Microsoft® Office Professional 97*).

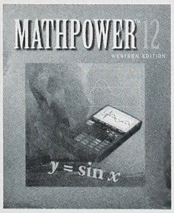
Note: Examples in this course show the TI-83 graphing calculator and the *Microsoft® Excel 97* spreadsheet program where applicable. If you have access to a videocassette recorder, you may wish to view the video *The TI-83 Graphing Calculator Video Tutor* to review some of the calculator's features. Check with your distance learning provider concerning the availability of this video.

Finally, you will need the following resources, which can be purchased locally:

- a mathematical instrument set (compass, protractor, ruler, and triangles)
- a notebook or binder in which to respond to the questions asked
- a notebook or binder for journal writing

Other Visual Cues

In addition to the Mathematical Process icon described earlier, you will find other visual cues throughout this course. Read the explanations given to discover what each icon prompts you to do.



Refer to the textbook.

View a videocassette.

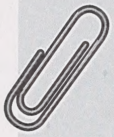
Use the Internet to explore a topic.

Answer the questions in the Assignment Booklet.

Use the Pure Mathematics 30 Companion CD.

Remember: *Any Internet website address given in this module is subject to change.*

CONTENTS



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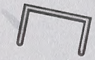

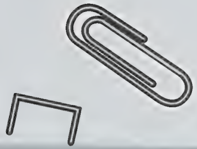

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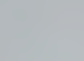


Module Overview

One of the most striking transformations in nature is the change of a caterpillar into an adult butterfly. In mathematics, change is also described as a transformation. Transformations in mathematics can be used to create and analyse patterns.

What do the wings of a swallowtail butterfly, the beadwork on Native Canadian clothing, the mosaic tiles of a Middle Eastern mosque, Scottish tartans, the geometrical designs on carpets and wallpaper, and the carved friezes of Greek temples have in common? Their patterns involve horizontal and vertical slides, reflections, rotations, expansions and compressions, and symmetries that can be described and classified mathematically.

In this module, you will investigate transformations of functions. The transformations will include horizontal and vertical slides, reflections, and expansions and compressions. In Section 1, you will explore each transformation separately; then, in Section 2, you will explore the effects of combining these transformations on the graphs of basic functions. Lastly, in Section 2, you will analyse the graphs of functions involving absolute values and reciprocals, working from simpler forms to more-complex forms.

Module 1: Transformations

Section 1: Transformations
Section 2: Graphing

Evaluation

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Transformations, has two sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

In this module, you will be directed to the accompanying Assignment Booklets. You are expected to complete two Assignment Booklets for this module. Your grading in this module is based upon the assignments you submit for evaluation. The mark distribution is as follows:

Assignment Booklet 1A	
Section 1 Assignment	**40 marks**
Assignment Booklet 1B	
Section 2 Assignment	**40 marks**
Final Module Assignment	**20 marks**
TOTAL	**100 marks**

Strategies for Completing a Module

Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklets, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 30.

Good luck!

Section 1

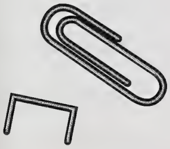
CORBIS/JAN BUTCHOFSKY-HOUSER

Transformations

Ancient builders and modern architects have used intricate tile patterns to adorn their work. If you study these patterns, you will observe that the artisans who laid the tiles often employed similar techniques. Their decorative motifs may include horizontal and vertical repetition, symmetry, or the gradual enlargement or reduction of a design. Did you know that many designers use graph paper or a grid to plan their layout before actually laying any tiles?

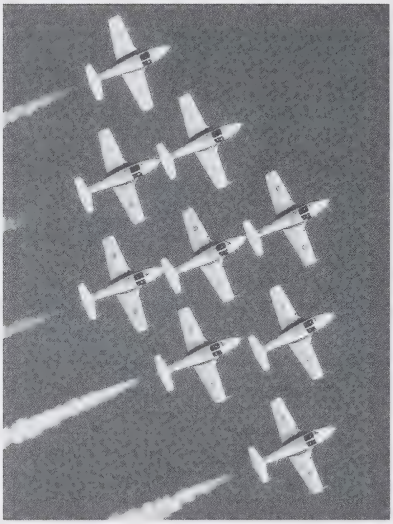
Graphs are also a way of representing and visualizing functions and relations.

In this section, you will create patterns that include horizontal and vertical repetitions, reflections, and expansions or compressions using functions and relations. You will then discover how to modify the original equations to produce these transformations. You will, of course, be using graph paper and a graphing calculator throughout your exploration.



Activity 1: Horizontal and Vertical Translations

Have you been fortunate enough to see the Snowbirds perform? In some of their routines, the pilots of the Canadian Force's aerobatics team manoeuver their jets so close together and fly with such precision that the planes appear to be a single aircraft.


1

If you have access to the Internet, you can find out more about the Snowbirds at the following website:

http://www.snowbirds.dnd.ca/

The formation in the photograph is called big diamond. Notice how it looks like a pattern that has been stamped in the sky by repositioning a single plane to the left, to the right, forwards, and backwards.

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

In mathematics, shifting or repositioning a graph in the coordinate plane is an example of a transformation—a change in the equation and graph of a function. How can you reposition the graphs of functions without changing their shape or size? What changes would you have to make to the equations of these functions? In this activity, you will explore translations—horizontal and vertical shifts of the graphs of functions.

a change in the equation and graph of a function

Turn to page 7 of *MATHPOWER™ 12* and read "Horizontal and Vertical Translations of Functions."

1. Answer the following questions on page 7 of the textbook:

   a. questions a. and b. of "Explore: Graph the Functions"
   b. questions 1 to 5 of "Inquire"

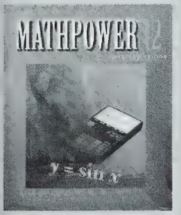
Turn to page 56 to compare your responses with those in the Appendix.

a transformation that produces a horizontal or vertical shift withou rotation

---

[1] Canadian Forces. Reproduced with permission of the Minister of Public Works and Government Services Canada, 2000.

In the preceding questions, you discovered that changing the value of *b* in the function $y = -5x^2 + b$ shifted the graph of $y = -5x^2$ vertically *b* units. Now, you will generalize this discovery for all functions.



Turn to page 8 of *MATHPOWER™ 12* and read from the top of the page to the red line on page 9, working through Examples 1 and 2.

2. Answer questions 1, 2, 21, and 22 of "Practice" on pages 10 and 11 of the textbook.

Turn to page 58 to compare your responses with those in the Appendix.

Vertical translations can be useful in visualizing the range of a function.

Mathematical Process

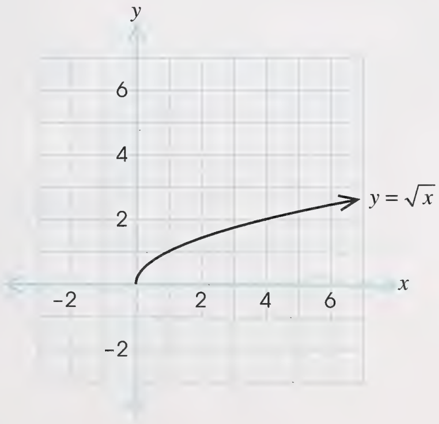
- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ☒ Visualization

Example

Compare the range of $y = \sqrt{x} + 3$ with the range of $y = \sqrt{x}$.

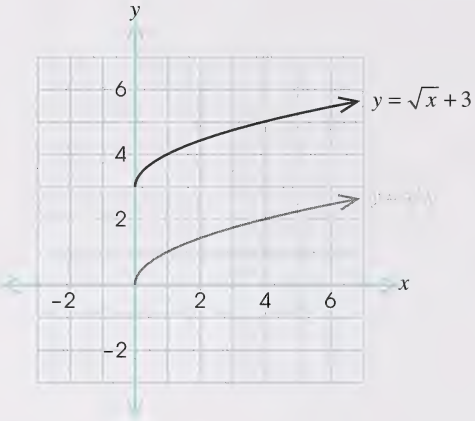
Solution

First, graph $y = \sqrt{x}$.

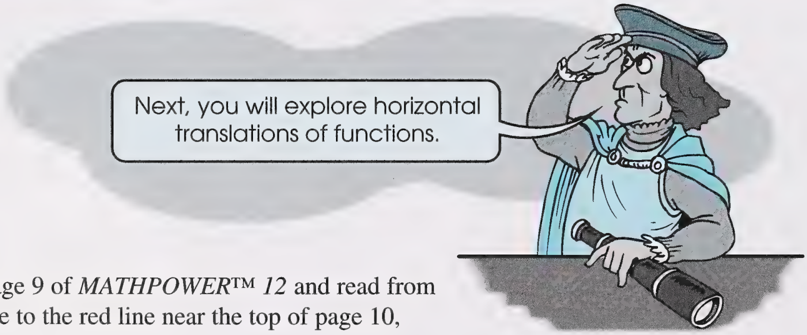


Because $\sqrt{x}$ is the principal or non-negative root of x, the range of $y=\sqrt{x}$ is $y\geq 0$.

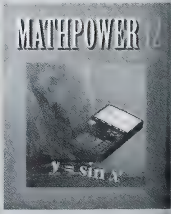
Next, graph $y=\sqrt{x}+3$. This is the graph of $y=\sqrt{x}$ translated 3 units up.



Because each y-value of $y=\sqrt{x}+3$ is 3 more than the corresponding y-value of $y=\sqrt{x}$, the range of $y=\sqrt{x}+3$ is $y\geq 0+3$ or $y\geq 3$.


Next, you will explore horizontal translations of functions.

Turn to page 9 of *MATHPOWER™ 12* and read from the red line to the red line near the top of page 10, working through Example 3.



3. Answer questions 3, 4, 25, and 26 of "Practice" on pages 10 and 11 of the textbook.

Turn to page 59 to compare your responses with those in the Appendix.

in an expression, a letter representing a constant that can be varied to produce other cases

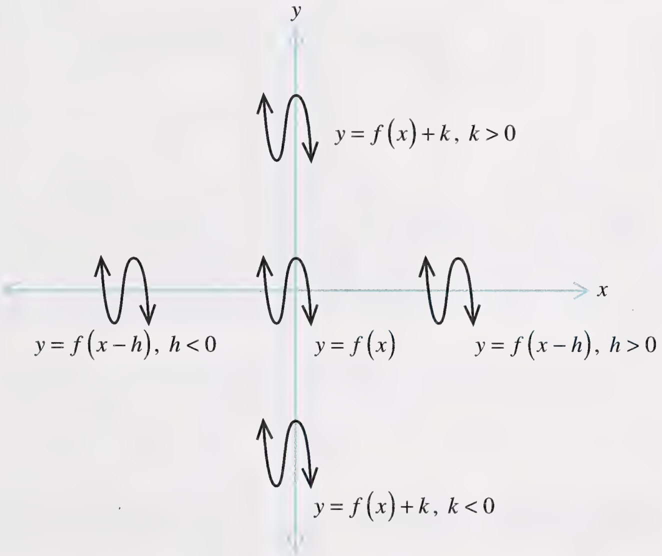
Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ☒ Visualization

You can summarize horizontal and vertical translations for any function. For the function $y=f(x)$ and parameters h and k, the following is true:

- If $h>0$, $y=f(x-h)$ moves the graph of $y=f(x)$ to the right h units.
- If $h<0$, $y=f(x-h)$ moves the graph of $y=f(x)$ to the left h units.
- If $k>0$, $y=f(x)+k$ moves the graph of $y=f(x)$ upward k units.
- If $k<0$, $y=f(x)+k$ moves the graph of $y=f(x)$ downward k units.

The following graph shows the effects of parameters h and k.



Next, you will investigate combinations of horizontal and vertical translations in detail.

Example

a. Graph $y = x^3$ on your graphing calculator.

b. Predict how the graph of $y = (x+4)^3 - 2$ is related to the graph of $y = x^3$.

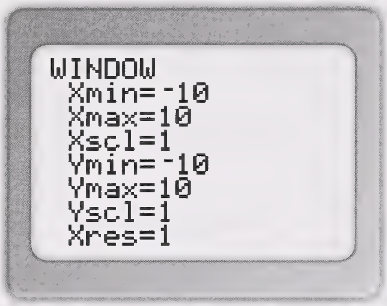
c. Confirm your prediction by graphing $y = (x+4)^3 - 2$.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Solution

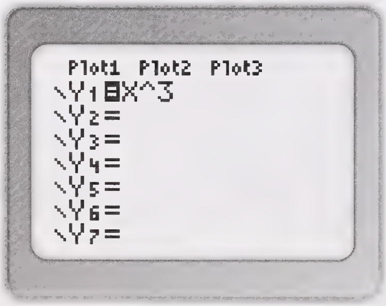
a. Use the standard window settings.



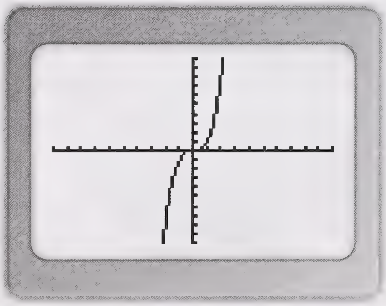
```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

Next, press the following to obtain the graph:

Y= X,T,Θ,n ^ 3 GRAPH

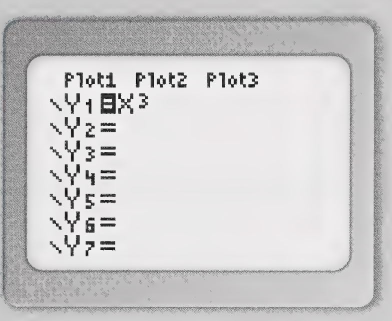


```
 Plot1 Plot2 Plot3
\Y1=X^3
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

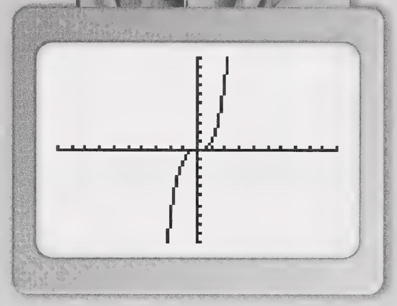


You can also enter $y = x^3$ on your graphing calculator another way.

Y= X,T,Θ,*n* MATH 3 GRAPH



```
 Plot1 Plot2 Plot3
\Y1■X³
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



b. The graph of $y=(x+4)^3-2$ and the graph of $y=x^3$ are congruent. To obtain the graph of $y=(x+4)^3-2$, you must translate the graph of $y=x^3$ horizontally and vertically.

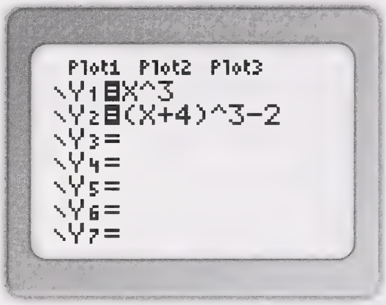
To determine these translations, find the values of parameters h and k in $y=f(x-h)+k$. Compare $y=(x+4)^3-2$ or $y=\left[x-(-4)\right]^3-2$ with $y=(x-h)^3+k$.

$\therefore\ h=-4$ and $k=-2$

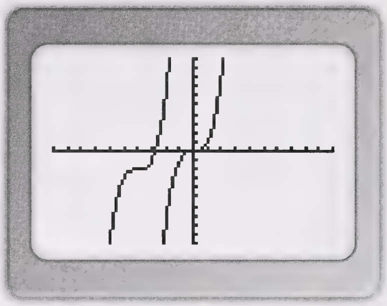
The graph of $y=(x+4)^3-2$ is the graph of $y=x^3$ translated 4 units to the left and 2 units down.

c. Graph $y=(x+4)^3-2$ on the same screen as $y=x^3$. Move the cursor to "Y_2 =," and press the following:

(X,T,Θ,*n* + 4) ^ 3 − 2 GRAPH



```
Plot1 Plot2 Plot3
\Y1=X^3
\Y2=(X+4)^3-2
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



Some students prefer to investigate horizontal and vertical translations by expressing the given function in the form $y-k=f(x-h)$. In this form, both the vertical and horizontal translations are identified in similar ways.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☐ Visualization

Consider the following example.

Example

a. Explain how the graphs of $y-3=|x-2|$ is related to the graph of $y=|x|$.

b. Explain how the graph of $y+3=|x+2|$ is related to the graph of $y=|x|$.

c. Graph all three functions in the same plane.

Solution

a. Determine the values of h and k from the form of the function, $y-k=f(x-h)$.

Compare $y-3=|x-2|$ with $y-k=|x-h|$.

$\therefore\ h=2$ and $k=3$

The graph of $y-3=|x-2|$ is the graph of $y=|x|$ translated 2 units to the right and 3 units up.

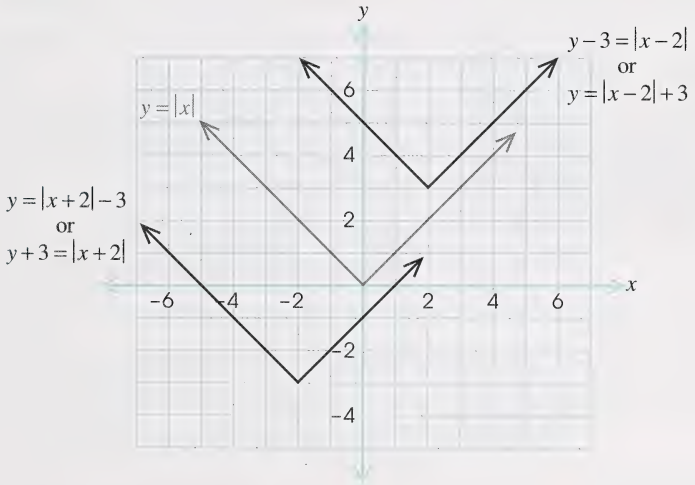
b. Determine the values of h and k from the form of the function, $y-k=f(x-h)$.

Compare $y+3=|x+2|$ or $y-(-3)=|x-(-2)|$ with $y-k=|x-h|$.

$\therefore\ h=-2$ and $k=-3$

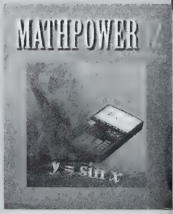
The graph of $y+3=|x+2|$ is the graph of $y=|x|$ translated 2 units to the left and 3 units down.

c.



Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ☒ Visualization



Turn to page 10 of *MATHPOWER™ 12* and work through Example 4.

4. Answer the following questions on pages 10 to 13 of the textbook:

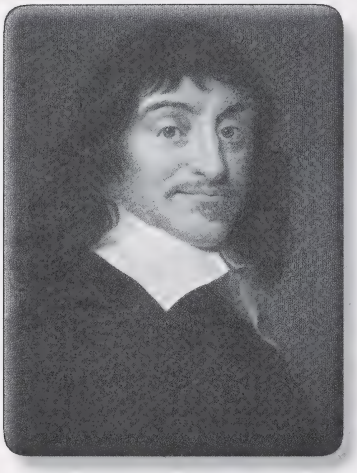
- **a.** questions 9, 10, 14, 15, 18.f., 27, 29, 30, and 33.f. of “Practice”
- **b.** questions 37, 42, and 44 of “Applications and Problem Solving”

Turn to page 61 to compare your responses with those in the Appendix.

Now Try This

Mathematical Process

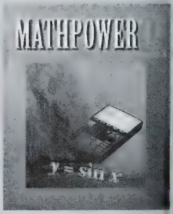
■ Communication
☒ Connection
■ Estimation
■ Mental Math
☒ Problem Solving
■ Reasoning
■ Technology
☒ Visualization


CORBIS/BETTMANN

A mathematician by the name of Rene Descartes (1596–1650) invented coordinate geometry in order to visualize functions and their relationships.

The horizontal and vertical translations of this activity are two ways of looking at the relationships among functions. Many relationships are made more apparent by graphs, diagrams, and sketches.

To solve the next problem, you will test your ability to analyse a diagram.



Turn to “PROBLEM SOLVING” on pages 14 and 15 of *MATHPOWER™ 12* and read “Use a Diagram.”

5. Answer question 2 of “Applications and Problem Solving” on page 15 of the textbook.

Turn to page 67 to compare your response with the one in the Appendix.

Looking Back

Mathematical Process

☒ Communication
☒ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

In this activity, you investigated horizontal and vertical translations of functions. As part of your investigation, you explored how to modify the equations of functions in order to translate their graphs horizontally and vertically.

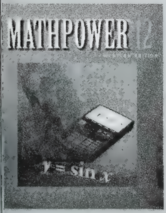
Can you translate French to English? Use your investigation skills to explore how the word “translate” used in this question is similar to the word “translate” used in mathematics. How is it different? Record your findings in your journal.

Activity 2: Reflections

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

reflection: an image produced as if in a mirror



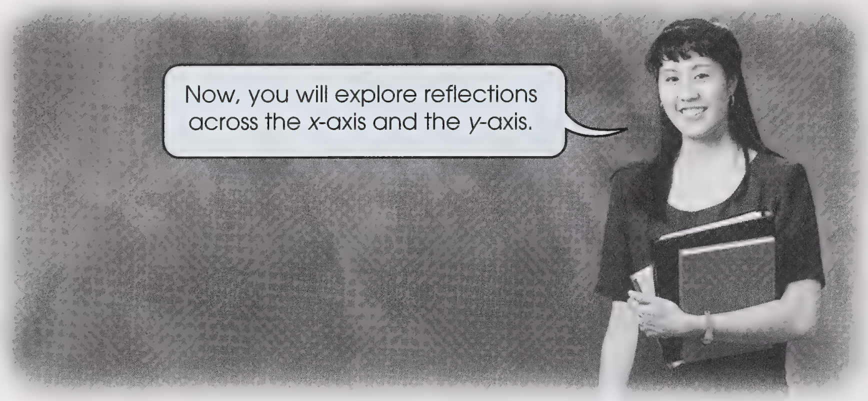
Reflections in water have long inspired artists and photographers. In the photograph, the iceberg and its reflection in the still arctic waters of Jones Sound (off the south coast of Ellesmere Island) combine to form an appealing symmetry that captivates an observer's imagination.

Reflections and symmetry also appeal to a mathematician's imagination, thus becoming a part of the analysis of functions and their graphs. In this activity, you will explore reflections in the x-axis, the y-axis, and across the line $y = x$.

Turn to page 16 of *MATHPOWER™ 12* and read "Reflections."

1. Answer the following questions on page 16 of the textbook:
    - **a.** "Explore: Graph the Functions"
    - **b.** questions 1 to 7 of "Inquire"

Turn to page 68 to compare your responses with those in the Appendix.


Now, you will explore reflections across the *x*-axis and the *y*-axis.

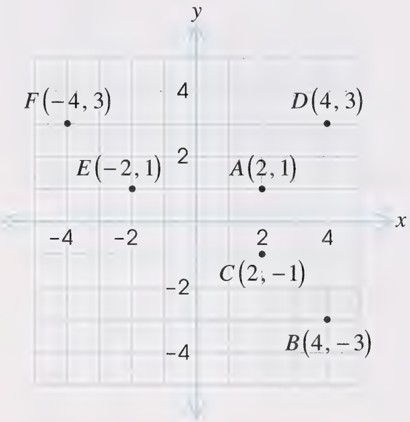
Example

Graph the points $A(2, 1)$, $B(4, -3)$, $C(2, -1)$, $D(4, 3)$, $E(-2, 1)$, and $F(-4, 3)$. Which pairs of points are mirror images in the *x*-axis? Which pairs of points are mirror images in the *y*-axis? Explain.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

Solution



Points *A* and *C*, as well as points *B* and *D*, are mirror images of each other in the *x*-axis. Notice that the *x*-coordinates in each pair are the same and the *y*-coordinates are opposites. Also, notice that if you joined the points, like *A* and *C*, the *x*-axis would bisect that segment at a right angle.

Points *A* and *E*, as well as points *D* and *F*, are mirror images in the *y*-axis. Notice this time that the *x*-coordinates in each pair are opposites and the *y*-coordinates are the same.

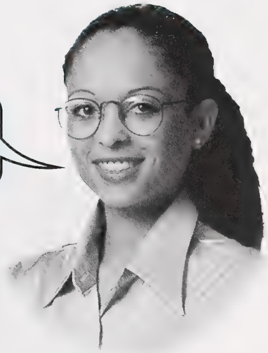
This is exactly what happens when you stand in front of a mirror; the line segment joining you and your image is bisected at right angles by the plane of the mirror.

Mathematical Process

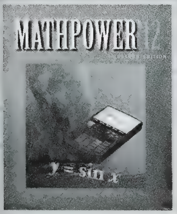
- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ☒ Reasoning
- ■ Technology
- ■ Visualization

You can generalize the results of the preceding example for any function, $y = f(x)$, as follows:

- Because (x, y) and $(x, -y)$ are **mirror images in the x-axis**, the graph of $y = -f(x)$ is the reflection of $y = f(x)$ in the x-axis.
- Because (x, y) and $(-x, y)$ are **mirror images in the y-axis**, the graph of $y = f(-x)$ is the reflection of $y = f(x)$ in the y-axis.



Now, you will apply these results to specific functions.



Turn to page 17 of *MATHPOWER™ 12* and read from the top of the page to Example 3 on page 18, working through Examples 1 and 2.

2. Answer the following questions on pages 25 to 27 of the textbook:

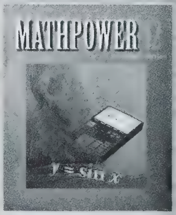
a. questions 2, 3, 7, 8, 13, and 14 of "Practice"
b. questions 32 and 34 of "Applications and Problem Solving"

Turn to page 71 to compare your responses with those in the Appendix.

You have explored reflections in the x-axis and reflections in the y-axis. A third type of transformation you will investigate is the reflection of a function across the line $y = x$. Remember, you explored functions and their inverses in Pure Mathematics 20. Recall that the graphs of a function, $y = f(x)$, and its inverse, $y = f^{-1}(x)$, are mirror images in the line $y = x$.

Review the procedures for finding and graphing inverses.

Turn to page 18 of *MATHPOWER*™ *12* and read from Example 3 to the bottom of page 19, working through Example 3.



3. Answer the following questions on pages 25 to 27 of the textbook:

  a. questions 5, 15, 16, 17, 18, and 26 of "Practice"
  b. questions 33 and 36 of "Applications and Problem Solving"

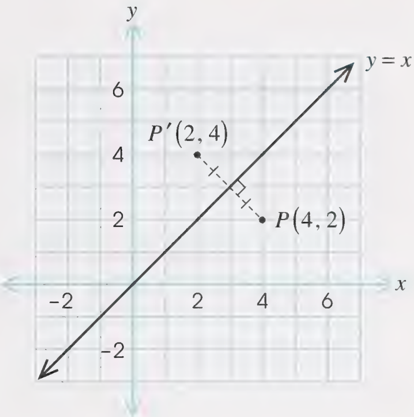
Turn to page 73 to compare your responses with those in the Appendix.

Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ☒ Reasoning
- ■ Technology
- ■ Visualization

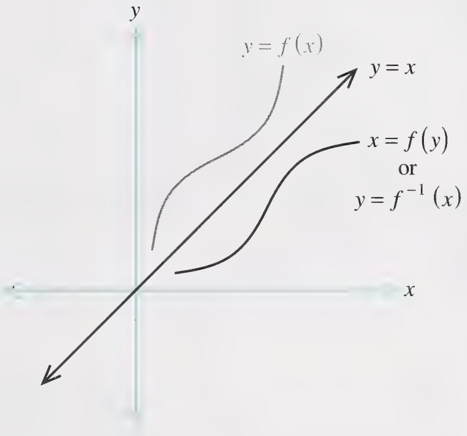
Recall that the inverse of an ordered pair, such as $P(4, 2)$, is formed by interchanging the coordinates of the ordered pair. Thus, the point $P'(2, 4)$ is the inverse of $P(4, 2)$. The segment PP' is bisected at a right angle by the line $y = x$. Therefore, points P and P' are reflections in the line $y = x$.

In general, (x, y) and its inverse, (y, x), are reflections in the line $y = x$.

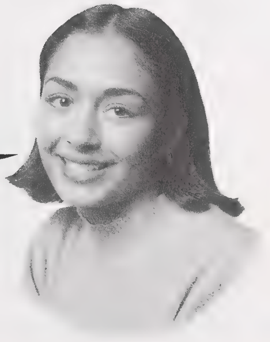


For functions, $y = f(x)$, recall that their inverses, $x = f(y)$, are also obtained by interchanging x and y. Therefore, as with ordered pairs, the graphs of $y = f(x)$ and $x = f(y)$ are reflections in the line $y = x$.

Note: If $x = f(y)$ is a function, it may be rewritten in functional notation as $y = f^{-1}(x)$.



You can use your graphing calculator to draw a function, $y = f(x)$, and its inverse, $x = f(y)$.



Example

Using your graphing calculator, graph $y = x^2 - 4$, its inverse, and $y = x$ on the same axes.

Mathematical Process

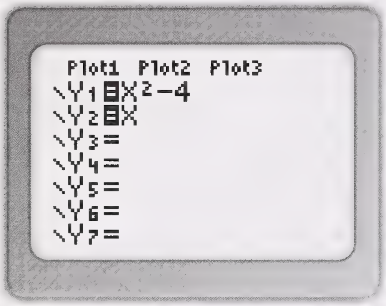
- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Solution

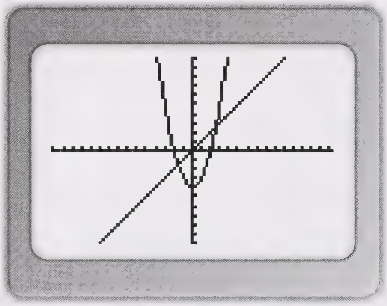
To set the window and to ensure that the line $y = x$ looks like it bisects the first and third quadrants, press ZOOM 6 ZOOM 5.

Graph $y = x^2 - 4$ and $y = x$ on your graphing calculator.

Y= X,T,Θ,n x^2 − 4 ENTER X,T,Θ,n GRAPH



```
Plot1 Plot2 Plot3
\Y1=X²-4
\Y2=X
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

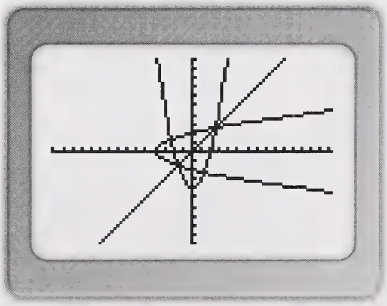


To graph the inverse of $y = x^2 - 4$, use the Draw Inverse feature (DrawInv) from the DRAW menu.

2nd [DRAW] 8 (8:DrawInv)

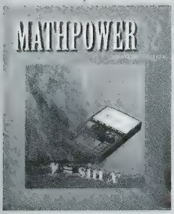
VARS [Select the Y-VARS menu.] 1

(1:Function…) 1 (1: Y_1) ENTER



Note: The equation $x = y^2 - 4$ is not a function.

You will now practise your skills using absolute value, quadratic, cubic, radical, and rational functions.



Turn to pages 20 to 24 of *MATHPOWER*™ *12* and work through Examples 4 to 8.

4. Answer the following questions on pages 27 and 28 of the textbook:

a. questions 25, 28, 29, and 31 of "Practice"
b. questions 39, 42, and 45 of "Applications and Problem Solving"

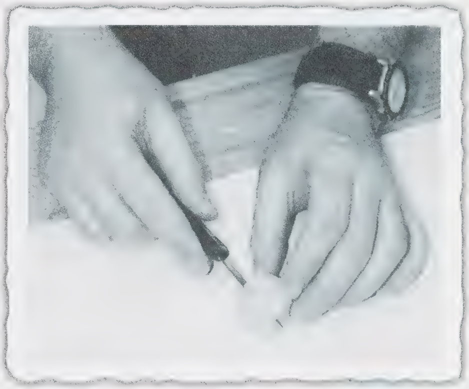
Turn to page 79 to compare your responses with those in the Appendix.

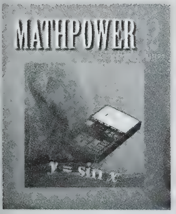
Now Try This

Mathematical Process
■ Communication
■ Connection
■ Estimation
■ Mental Math
☒ Problem Solving
■ Reasoning
■ Technology
☒ Visualization

You have just examined patterns formed by reflecting the graphs of functions. The search for a pattern and its description is fundamental to all branches of mathematics.

In the next question, you will investigate a particular pattern produced by a plane intersecting a cube.





5. Turn to page 15 of *MATHPOWER*™ *12* and answer question 5 of "Applications and Problem Solving."

Turn to page 84 to compare your response with the one in the Appendix.

Looking Back

Mathematical Process
☒ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

In this activity, you explored reflections of functions in the *x*-axis, in the *y*-axis, and in the line $y = x$.

In your journal, summarize these reflections; then illustrate each summary statement with an example. Exchange summaries with another student taking Pure Mathematics 30. Critique each other's work, and revise as necessary.

Activity 3: Stretches of Functions

Mathematical Process

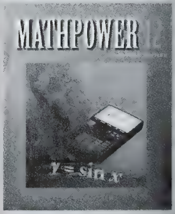
- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Many people, young and old, are fascinated by their reflections in curved mirrors. Unlike their images in flat mirrors, the faces that stare back at them may be stretched in one direction and compressed in another, producing unusual and amusing effects. Of course, curved mirrors serve many practical uses as well. Rearview mirrors on vehicles, traffic mirrors at blind intersections, and store security mirrors are curved to create a wider field of vision for safety and security.

CORBIS/JEREMY HORNER

In this activity, you will investigate transformations that expand or compress graphs vertically and transformations that expand or compress graphs horizontally. Later, you will combine vertical and horizontal stretches with reflections.

Turn to page 30 of *MATHPOWER*™ *12* and read "Stretches of Functions."

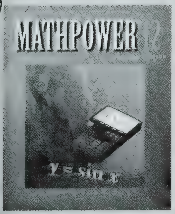


1. Answer the following questions on page 30 of the textbook:

   a. "Explore: Graph the Functions"
   b. questions 1 to 4 of "Inquire"

Turn to page 85 to compare your responses with those in the Appendix.

The first set of transformations you will explore are vertical stretches. Some will expand the graphs; others will compress them.



Turn to page 31 of *MATHPOWER™ 12* and read from the top of the page to the red line on page 33, working through Examples 1 to 3.

2. Answer the following questions on pages 38 to 40 of the textbook:

- **a.** questions 1, 2, 7, 13, 15, 16, 20, 21, 33, 34, 35, and 38 of "Practice"
- **b.** questions 43, 45, 50.c., and 51.a. of "Applications and Problem Solving"

Turn to page 86 to compare your responses with those in the Appendix.

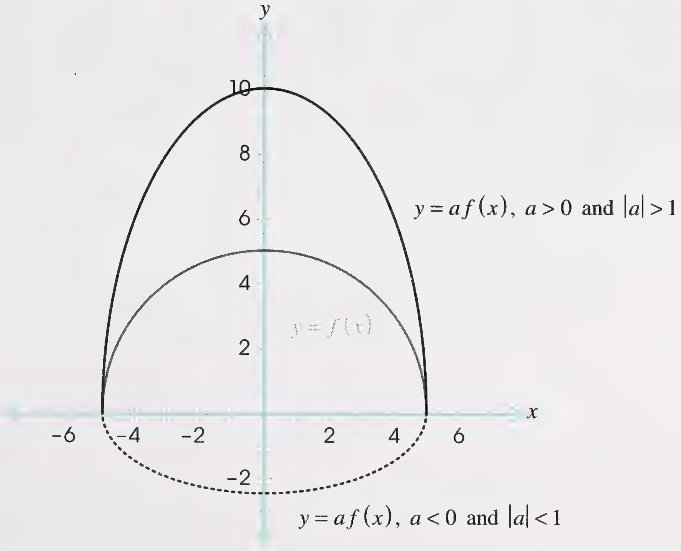
You have discovered that the graph of $y = af(x)$ can be obtained by multiplying the y-values of points on the graph of $y = f(x)$ by a factor of a. This means a point (x, y) on the graph of $y = f(x)$ becomes (x, ay) on the graph of $y = af(x)$. You can summarize these observations as follows:

Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
☒ Reasoning
■ Technology
☒ Visualization

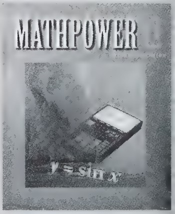
- If $|a| > 1$, the graph is expanded vertically.
- If $0 < |a| < 1$, the graph is compressed vertically.
- If $a < 0$, the graph is reflected in the x-axis.
- If $a \neq 0$, the graphs of $y = f(x)$ and $y = af(x)$ have the same x-intercepts.

The following sketch illustrates the preceding statements.



Now, you will investigate horizontal stretches.

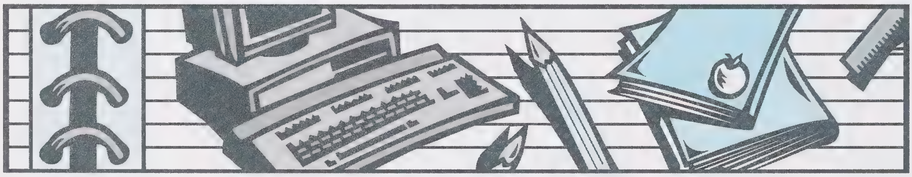
Turn to page 33 of *MATHPOWER*™ *12* and read from the red line to the bottom of page 36, working through Examples 4 to 6.



3. Answer the following questions on pages 38 to 40 of the textbook:

- **a.** questions 3, 4, 9, 10, 18, 24, 25, and 41.b. of "Practice"
- **b.** questions 46, 47, and 51.c. of "Applications and Problem Solving"

Turn to page 93 to compare your responses with those in the Appendix.



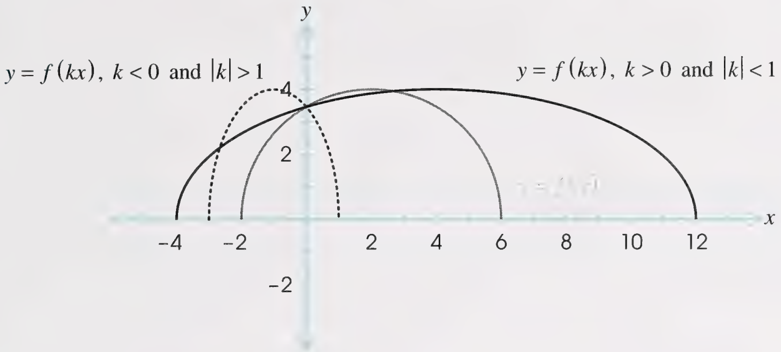
You have discovered that the graph of $y=f(kx)$ can be obtained by dividing the x-values of points on the graph of $y=f(x)$ by k. This means a point (x, y) on the graph of $y=f(x)$ becomes $\left(\frac{x}{k}, y\right)$ on the graph of $y=f(kx)$. You can summarize these observations as follows:

- If $|k|>1$, the graph is compressed horizontally.
- If $0<|k|<1$, the graph is expanded horizontally.
- If $k<0$, the graph is reflected in the y-axis.
- If $k\neq 0$, the graphs of $y=f(x)$ and $y=f(kx)$ have the same y-intercept.

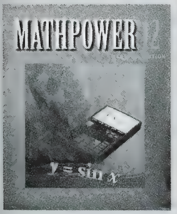
Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☒ Visualization

The following sketch illustrates the preceding statements.



Now, work through some examples involving both horizontal and vertical stretches.



Turn to pages 37 and 38 of *MATHPOWER™ 12* and work through Examples 7 and 8.

4. Answer the following questions on pages 38 to 40 of the textbook:

a. questions 5, 11, 27, 29, 32, and 41.e. of "Practice"
b. questions 48 and 49 of "Applications and Problem Solving"

Turn to page 97 to compare your responses with those in the Appendix.

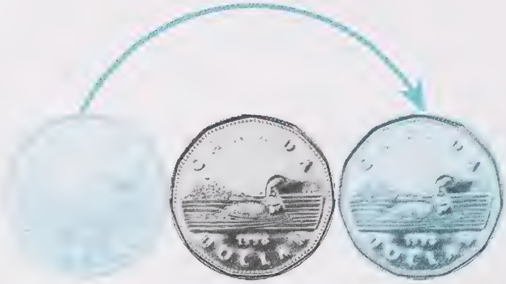
Now Try This

So far, you have examined transformations involving horizontal and vertical translations and stretches. The following transformation involves a rotation that may stretch your problem-solving skills.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

5. Two loonies are placed side by side on a flat surface. The loonie on the left is then rolled around the other loonie, without slipping, until it is on the right of the second loonie (as shown in the diagram). Once on the right side, the first loonie is in the same upright position as it was when it was on the left side. Since the first loonie has rolled only halfway around the second loonie, why isn't the first loonie upside down?



Turn to page 99 to compare your response with the one in the Appendix.

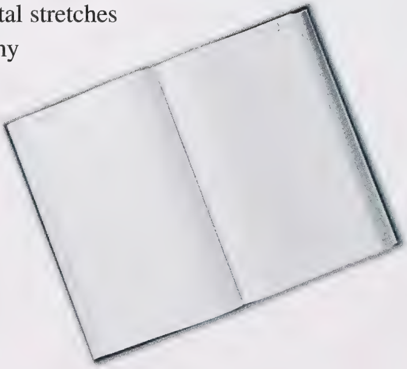
Looking Back

Mathematical Process

- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

In this activity, you explored vertical and horizontal stretches and examined how to transform the equation of any function, $y = f(x)$, to produce these stretches.

In your journal, explain how vertical and horizontal stretches are similar and how they are different. Illustrate using examples.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Often, it is difficult to visualize the graphs of relations in the form $x = f(y)$. Is there an easy way of graphing relations like $x = y^2$, $x = y^3 - y^2$, and $x = |y|$?

Recall, from Activity 2, that the relation $x = f(y)$ is the inverse of the function $y = f(x)$. You will use the concept of inverses in the next example.

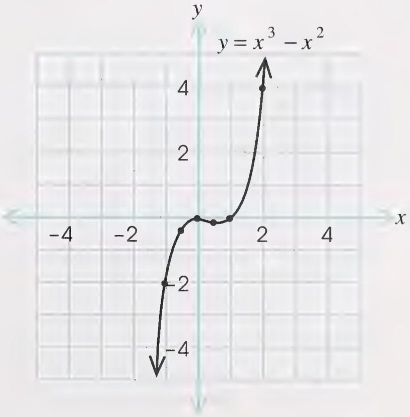
Example

Draw the graph of $x = y^3 - y^2$.

Solution

It is easier to think in terms of $y = f(x)$, its inverse; so, start by drawing the graph of $y = x^3 - x^2$.

x	y
-1	-2
-0.5	-0.375
0	0
0.5	0.125
1	0
2	4



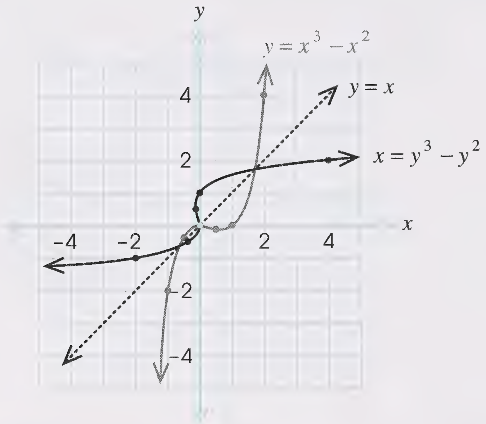
Next, reflect this graph across the line $y = x$ to obtain the graph of $x = y^3 - y^2$. To do this, select points from $y = x^3 - x^2$ and plot their inverses.

$y = x^3 - x^2$

x	y
-1	-2
-0.5	-0.375
0	0
0.5	0.125
1	0
2	4

$x = y^3 - y^2$

x	y
-2	-1
-0.375	-0.5
0	0
0.125	0.5
0	1
4	2



Use the approach from the preceding example to draw each of the following curves:

1. $x = y^2$

2. $x = |y|$

Turn to page 100 to compare your responses with those in the Appendix.

Enrichment

Symmetry is common in nature. The human body, a bird in flight, and certain plants and flowers can all be visually split in two, where one side mirrors the other.

In mathematics, it is to your advantage to recognize symmetries whenever they occur. For example, to graph certain functions and relations, it may only be necessary to sketch a portion of the curve, the remainder being reflections of the portions already known.

Consider the following example.

Example

Reflect $f(x)=x^2+2$ in the y-axis.

Mathematical Process

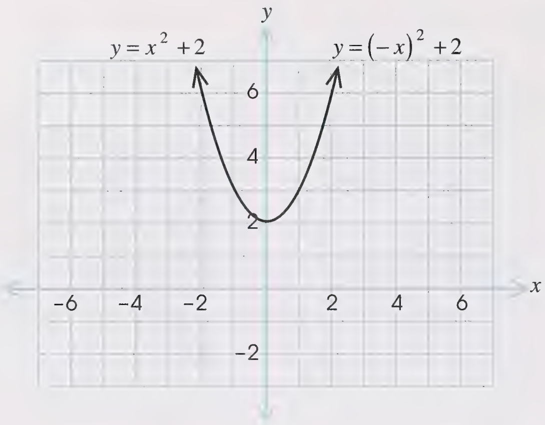
- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ☒ Visualization

Solution

In Activity 2, you discovered that $y=f(-x)$ is the reflection of $y=f(x)$ in the y-axis. Therefore, substitute $-x$ for x.

$$\begin{aligned} f(x)&=x^2+2\\ f(-x)&=(-x)^2+2\\ &=x^2+2\\ &=f(x) \end{aligned}$$

Because $f(-x)=f(x)$, the function $f(x)=x^2+2$ is its own mirror image in the y-axis.



Therefore, $f(x)=x^2+2$ is symmetric with respect to the y-axis.

If $f(-x)=f(x)$ for any function, $y=f(x)$, the graph is symmetric with respect to the y-axis. Do the following questions.

1. Determine which of the following graphs are symmetric with respect to the y-axis.

a. $f(x)=|x|$

b. $f(x)=-x^4-5x^2$

c. $f(x)=x^3$

2. Each of the following graphs has a vertical axis of symmetry. Use your knowledge of transformations to find the equation of the axis of symmetry in each case.

a. $f(x)=2(x-4)^2$

b. $y=-|x+1|$

Turn to page 101 to compare your responses with those in the Appendix.

Now, investigate graphs that are symmetric with respect to the *x*-axis.

Example

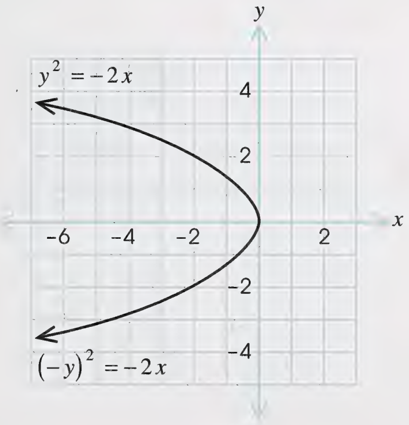
Reflect $y^2 = -2x$ in the *x*-axis.

Solution

Recall that $y = -f(x)$, or equivalently $-y = f(x)$, is the mirror image of $y = f(x)$ in the *x*-axis. Therefore, substitute $-y$ for *y*.

$$y^2 = -2x$$
$$(-y)^2 = -2x$$
$$y^2 = -2x$$

Because the resulting relation is identical to the original relation, $y^2 = -2x$ is its own mirror image in the *x*-axis.



Therefore, $y^2 = -2x$ is symmetric with respect to the *x*-axis.

A curve is symmetric with respect to the x-axis if the equation of that curve is unchanged when y is replaced by $-y$. Do the following questions.

3. Determine which of the following graphs are symmetric with respect to the x-axis.

a. $x = |y|$

b. $y^2 = x^2$

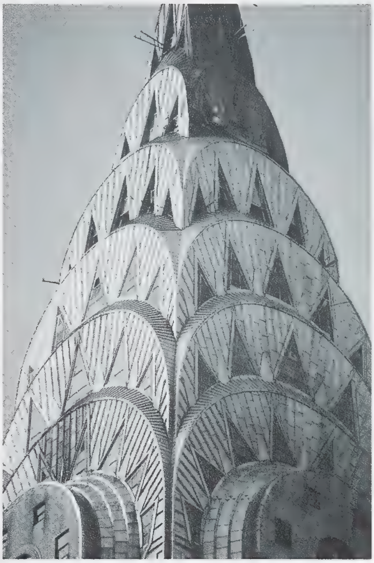
c. $y = x^4 - 1$

d. $y^2 = x - 2$

Turn to page 103 to compare your responses with those in the Appendix.

Conclusion

In this section, you examined transformations of functions. You translated the graphs of functions vertically and horizontally; you reflected graphs of functions in the x-axis, in the y-axis, and in the line $y = x$; and you stretched graphs of functions vertically and horizontally.



Many architects and designers employ lateral and vertical shifts of patterns, reflections and symmetry, and vertical or horizontal compressions or expansions to create novel and appealing designs.

An architect by the name of William van Alen incorporated these mathematical concepts in his design of the Chrysler Building, located in New York. Completed in 1930, the 77-storey building became the city's tallest building with a height of 319 m. Today, it is the fourth largest building in New York, following the two World Trade Centre buildings (417 m) and the Empire State Building (381 m). The Chrysler Building, though, is still considered by many as the most beautiful building in New York.

If you have access to the Internet, you can obtain more information about the Chrysler Building at the following website:

http://www.luciddreams.com/chryslerbuilding/frames.html

Assignment

Turn to Assignment Booklet 1A and complete the assignment for Section 1. Remember to submit your completed Assignment Booklet for assessment before starting Section 2.

Section 2

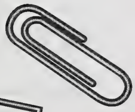
CORBIS

Graphing

Have you noticed how the designs of a variety of everyday objects contain mathematical characteristics? The geometric patterns in carpets, tiles, friezes, wallpaper, fabrics, and clothing often contain many of the same themes. Translations, reflections, and rotations are a few of the strategies designers use to create a pleasing overall effect.

Some of the traditional designs are readily identifiable as belonging to a particular group of people or culture. Others may be more modern. However, all designs conform to basic mathematical rules.

In this section, you will explore some of the rules that govern graphing functions. You will discover how complicated graphs can be obtained from simpler, basic forms; you will investigate combinations of translations, reflections, dilations, and compressions; and you will work from the graphs of familiar functions in order to understand the effects of absolute value and the reciprocal.



Activity 1: Combinations of Transformations

The butterfly is one of nature's marvels. This insect is transformed three times: from egg to larva (or caterpillar), from larva to pupa, and from pupa to adult (or butterfly).

The wings of many species of butterflies are brilliantly marked and coloured, where the pattern on one wing mirrors the pattern on the other. Also, each wing is covered with scales, which contain parallel ridges that act as a series of prisms transforming the reflected light into its constituent colours. On a number of species, a thin film also covers the scales. This film, like a film of oil on water, scatters colour in a blaze of iridescence when struck by sunlight.

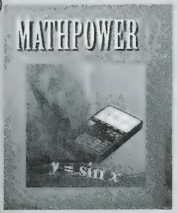
If you have access to the Internet, you can explore the world of the butterfly at the following website:

http://butterflywebsite.com

Understanding the transformations of the butterfly helps you appreciate their place in nature. Likewise, exploring the transformations of functions will enhance your understanding of functions and their role in mathematics and how transformations can be used to model real-world situations. In particular, in this activity, you will investigate how transformations can be combined to create intricate graphs from simpler forms.

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization



Turn to page 44 of *MATHPOWER™ 12* and read "Combinations of Transformations."

1. Answer the following questions on page 44 of the textbook:

- **a.** questions a. and b. of "Explore: Graph the Functions"
- **b.** questions 1 to 7 of "Inquire"

Turn to page 104 to compare your responses with those in the Appendix.

In the next example, you are going to combine a stretch, a reflection, and a horizontal and vertical translation to transform a graph.

Example

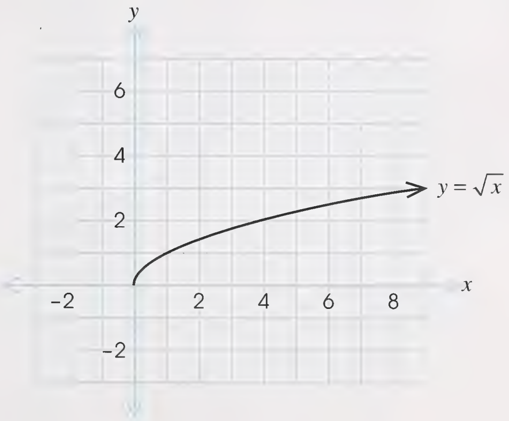
Show, using a series of transformations, how the graph of $y = -3\sqrt{x-2} + 4$ can be obtained from the graph of $y = \sqrt{x}$.

Solution

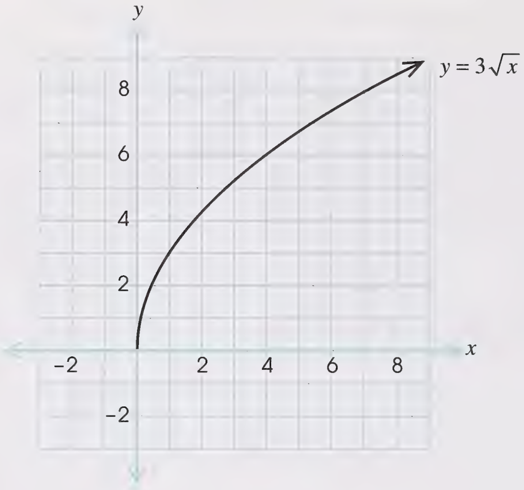
Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☒ Visualization

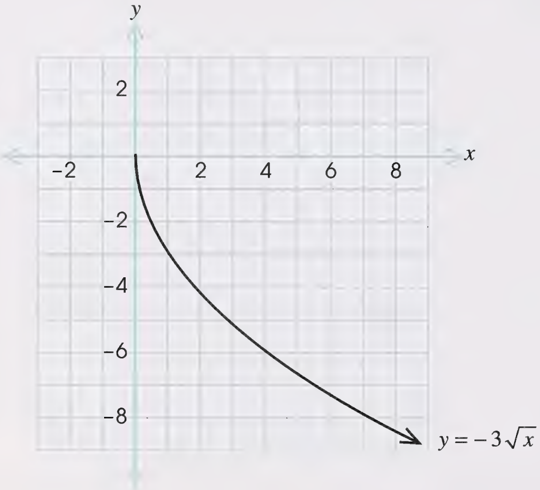
Step 1: Draw the original graph.



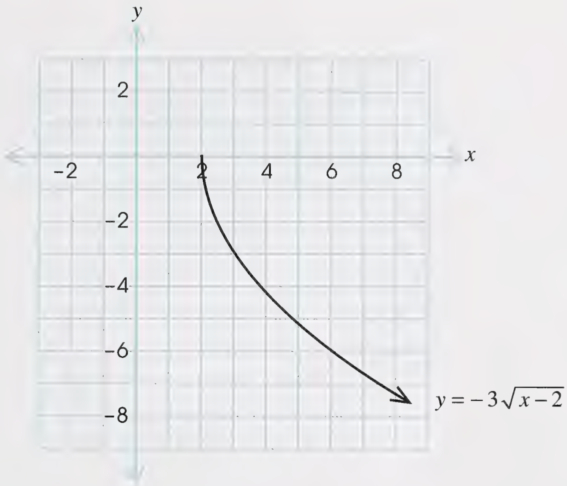
Step 2: Expand the graph vertically by a factor of 3.



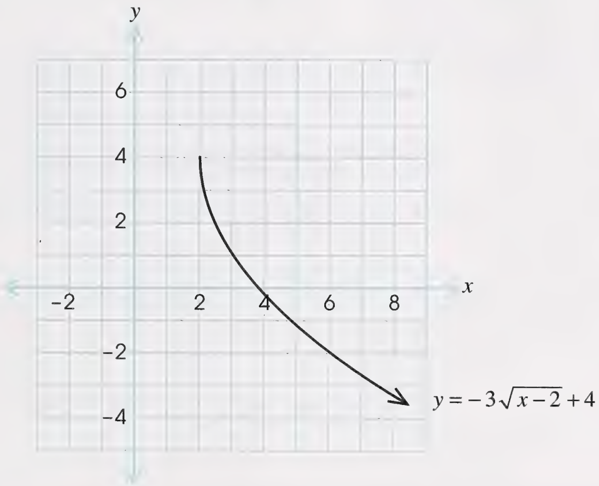
Step 3: Reflect the graph in the *x*-axis.



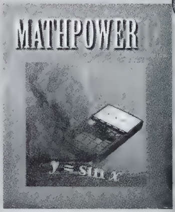
Step 4: Translate the graph 2 units to the right.



Step 5: Translate the graph 4 units up.



Turn to page 45 of *MATHPOWER™ 12* and read from the top of the page to the bottom of page 46, working through Examples 1 to 4.



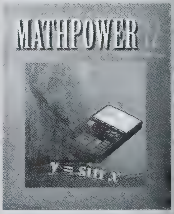
2. Answer the following questions on pages 47 to 49 of the textbook:

- **a.** questions 1, 5, 9, 14, 18, 19.a., 23, 31, 32, and 39 of "Practice"
- **b.** questions 49, 50, and 56 of "Applications and Problem Solving"

Turn to page 107 to compare your responses with those in the Appendix.

In the first part of this activity, you used transformations identified from the equations of functions to draw their graphs. Now, you will work backwards—finding the equations of graphs that have been transformed.

Turn to page 47 of *MATHPOWER™ 12* and work through Example 5.



3. Answer the following questions on pages 48 and 49 of the textbook:

- **a.** questions 44, 45, and 47 of "Practice"
- **b.** questions 52.a. and 54 of "Applications and Problem Solving"

Turn to page 111 to compare your responses with those in the Appendix.

Mathematical Process

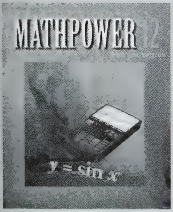
- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Now Try This

Are you interested in a career as a doctor of veterinary medicine? In 1969, the Western College of Veterinary Medicine in Saskatoon, Saskatchewan, was established to serve students across Western Canada.

If you have access to the Internet, you can research more about the Western College of Veterinary Medicine and about veterinary medicine in general at the following website:

http://www.usask.ca/wcvm/index.html



Turn to "CAREER CONNECTION" on page 60 of *MATHPOWER™ 12* and read "Veterinary Medicine."

4. Answer questions 1 to 6 of Investigation 1, "Ages of Cats and Dogs" on page 60 of the textbook.

Turn to page 114 to compare your responses with those in the Appendix.

Looking Back

In this activity, you explored combinations of transformations. Specifically, you combined vertical and horizontal translations, horizontal and vertical expansions and compressions, and reflections across each axis.

Mathematical Process

- ☒ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization

Transformations can be thought of as operations similar to the basic operations of addition, subtraction, multiplication, division, and finding powers and roots. You know that the arithmetic operations must be performed in a particular order. What about transformations? They are operations on functions. Should transformations be performed in a particular order? Research this question, and summarize your findings in your journal.

Activity 2: Reciprocal and Absolute Value Functions

Mathematical Process

- ■ Communication
- ☒ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization

Have you stood by the side of a highway and noticed how the sound of the engine of an approaching truck seems higher in pitch than when it passes you and pulls away? Similarly, have you heard a change in the wail of a police siren as the patrol car passes you?

The frequency, f, of the sound you hear from a source of frequency, f_s, as it moves away from you at velocity, v (in m/s), can be approximated by the *function*

$$f = \frac{f_s}{1 + 0.003v}$$

For example, suppose a siren emitting a sound of 1000 Hz (cycles/second) is coming from a police car racing away from you at 30 m/s. The frequency of sound you hear is

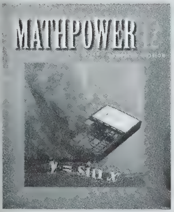
$$\begin{aligned} f &= \frac{f_s}{1 + 0.003v} \\ &= \frac{1000}{1 + 0.003(30)} \\ &\doteq 917 \text{ Hz} \end{aligned}$$

The function $f = \frac{f_s}{1+0.003v}$ is an example of a reciprocal function, a function obtained by taking the reciprocal of the function $y = f(x)$.

a function of the form $y = \frac{1}{f(x)}$

In this activity, you will analyse the relationship between the graph of a reciprocal function, $y = \frac{1}{f(x)}$, and the graph of $y = f(x)$. You will then investigate the relationship between the graph of an absolute value function, $y = |f(x)|$, and the graph of $y = f(x)$.

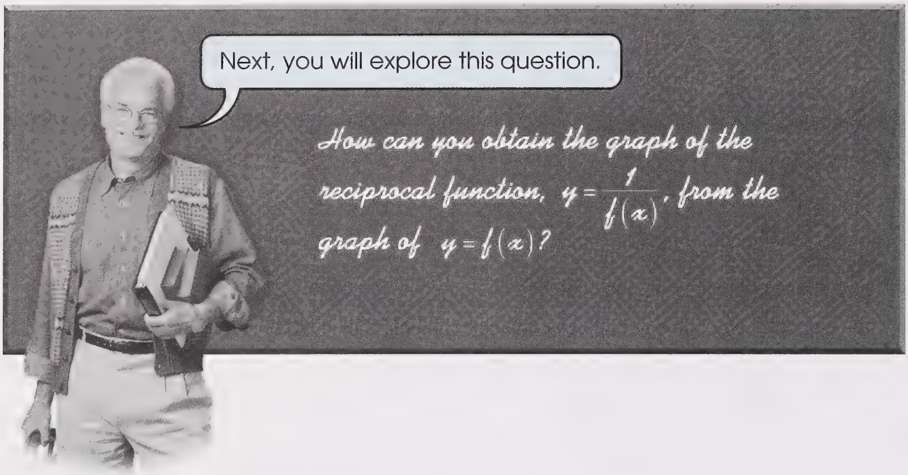
a function of the form $y = |f(x)|$



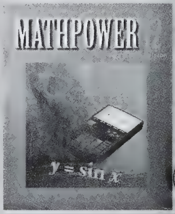
Turn to page 50 of *MATHPOWER™ 12* and read "Reciprocal Functions and Absolute Value Functions."

1. Answer the following questions on pages 50 and 51 of the textbook:
   - **a.** questions a. to d. of "Explore: Graph the Function"
   - **b.** questions 1 to 7 of "Inquire"

Turn to page 116 to compare your responses with those in the Appendix.


Next, you will explore this question.
How can you obtain the graph of the reciprocal function, $y = \frac{1}{f(x)}$, from the graph of $y = f(x)$?

Turn to page 51 of *MATHPOWER™ 12* and read from the red line to the bottom of page 53, working through Examples 1 to 3.



2. Answer the following questions on pages 56 and 57 of the textbook:

- **a.** questions 3, 7, 10, 13, 17, and 19 of "Practice"
- **b.** questions 44 and 54 of "Applications and Problem Solving"

Turn to page 118 to compare your responses with those in the Appendix.

Did you know that you can use your graphing calculator to graph functions and their reciprocals without having to enter the reciprocal function directly? Work through the next example.

Mathematical Process

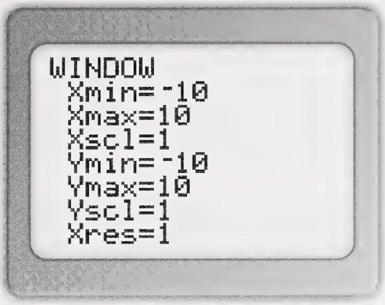
- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ☒ Technology
- ■ Visualization

Example

Graph $y = x^2 + 0.5$ and its reciprocal, $y = \frac{1}{x^2 + 0.5}$, on your graphing calculator.

Solution

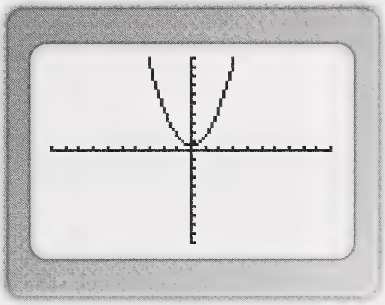
Begin by using the standard window setting.



```
WINDOW
 Xmin=⁻10
 Xmax=10
 Xscl=1
 Ymin=⁻10
 Ymax=10
 Yscl=1
 Xres=1
```

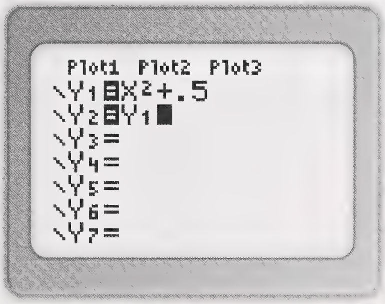
Next, enter the original function, $y = x^2 + 0.5$, as function Y_1.

Y= X,T,Θ,n x^2 + . 5 GRAPH



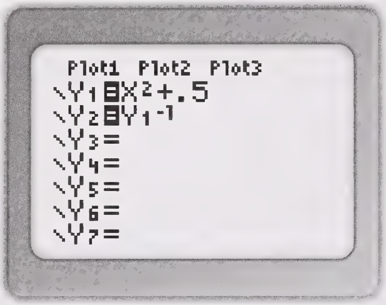
To enter the reciprocal function, paste "Y_1" beside "Y_2 =."

Y= VARS [Select the Y-VARS menu.] 1 (1:Function…) 1 (1: Y_1)



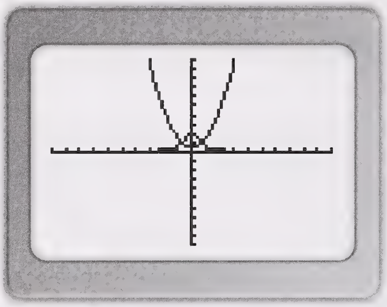
```
 Plot1  Plot2  Plot3
\Y1=X²+.5
\Y2=Y1█
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Next, press x^{-1} to form the reciprocal.



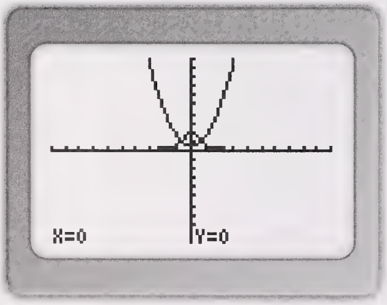
```
Plot1 Plot2 Plot3
\Y1=X²+.5
\Y2=Y1⁻¹
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Press GRAPH to see both graphs.

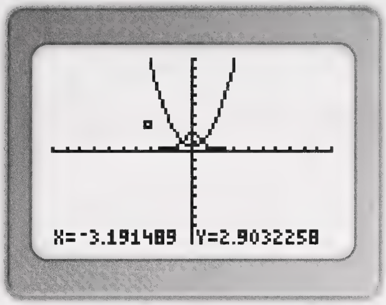


The graph of the reciprocal function in the preceding example appears small on the screen. Using the Zoom feature, you can expand the portion of the graph near the origin.

Press ZOOM 1 (1:Zbox).

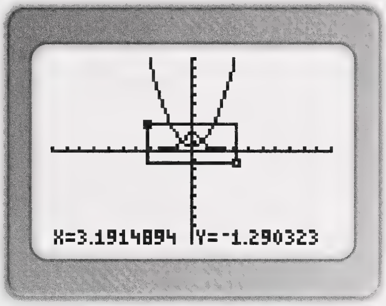


The cursor is located at the origin. Use the arrow keys to position the upper-left corner of the new viewing window; then press ENTER.



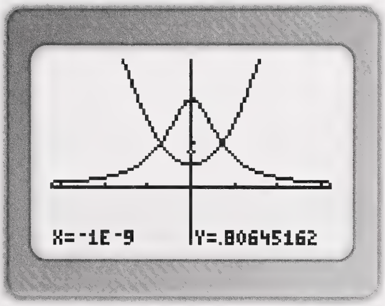
```
X=-3.191489  Y=2.9032258
```

Now, use the arrow keys to position the lower-right corner of the new viewing window.



```
X=3.1914894  Y=-1.290323
```

Press ENTER to display the graphs in the new window.

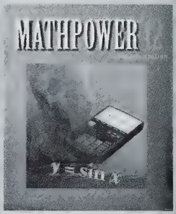


```
X=-1E-9      Y=.80645162
```

Note: To view a new pair of functions, simply change Y_1. You do not have to change Y_2 since it is expressed in terms of Y_1.

In the next part of this activity, you will investigate how to obtain the graph of the absolute value function, $y = |f(x)|$, from the graph of $y = f(x)$.

Read pages 54 and 55 of *MATHPOWER™ 12*, working through Examples 4 to 7.



3. Answer the following questions on pages 56 and 57 of the textbook:

a. questions 28, 32, 34, and 41 of "Practice"
b. questions 45, 47, 50, and 52 of "Applications and Problem Solving"

Turn to page 122 to compare your responses with those in the Appendix.

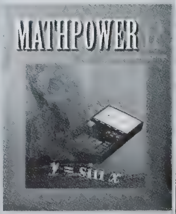
Now Try This

Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ☒ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ☒ Visualization

Now that you have practised combinations of transformations on a variety of functions, you should be able to visualize and perform simple transformations mentally, not having to resort to pencil and paper. The following questions will hone your mental skills.

4. Turn to page 3 of *MATHPOWER™ 12* and answer questions 1 to 12 of "Mental Math."



Turn to page 127 to compare your responses with those in the Appendix.

Mathematical Process

- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Looking Back

In this activity, you compared the graphs of reciprocal functions and absolute value functions with the original functions from which they were formed.

In your journal, summarize how you can obtain the graphs of $y=\frac{1}{f(x)}$ and $y=|f(x)|$ from the graph of $y=f(x)$. Illustrate your summaries with examples. Then share your summaries with another student taking Pure Mathematics 30.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In Activity 2, you graphed a function and its reciprocal using a graphing calculator. You can also use your graphing calculator to help analyse absolute value functions.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

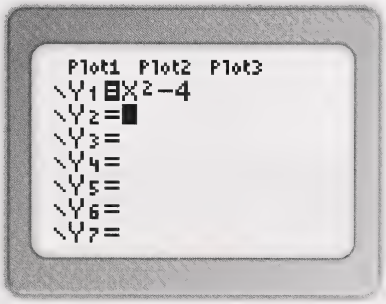
Example

Graph $y=x^2-4$ and $y=|x^2-4|$ on your graphing calculator.

Solution

Using the standard window settings, enter the first function, $y=x^2-4$, as function Y_1.

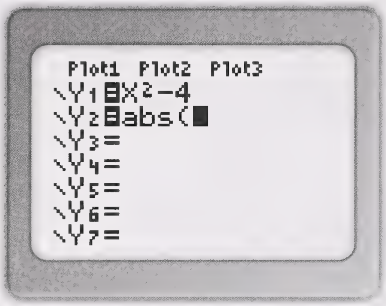
Y= X,T,Θ,*n* x^2 − 4 ENTER



```
 Plot1  Plot2  Plot3
\Y1■X²-4
\Y2=█
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Now, use the Absolute Value feature to define function Y_2.

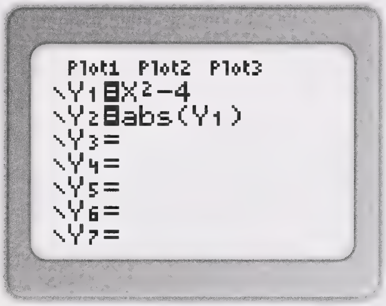
MATH [Select the NUM menu.] 1 (1:abs( )



```
Plot1 Plot2 Plot3
\Y1=X²-4
\Y2=abs(
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

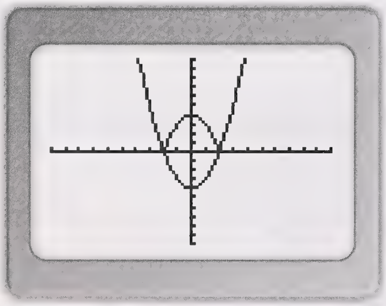
Next, paste "Y_1" after "abs(."

VARS [Select the Y-VARS menu.] 1 (1:Function…) 1 (1: Y_1))



```
Plot1 Plot2 Plot3
\Y1=X²-4
\Y2=abs(Y1)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Press GRAPH to see both graphs.



Now, you can enter different functions for Y_1 and graph $y = f(x)$ and $y = |f(x)|$. Simply press GRAPH after entering Y_1, leaving Y_2 as defined in the example.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Use the method from the preceding example to graph each of the following functions and its absolute value on your graphing calculator.

1. $y = x(x-2)(x+2)$

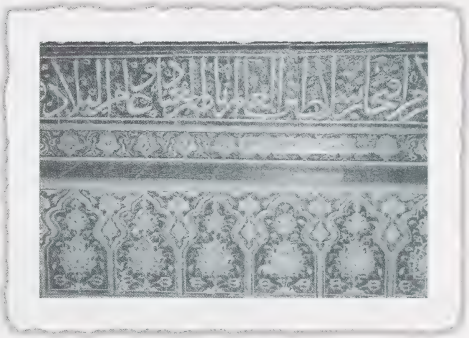
2. $y = -x^2 - 2$

3. $y = 2^x - 2$

Turn to page 127 to compare your responses with those in the Appendix.

Enrichment

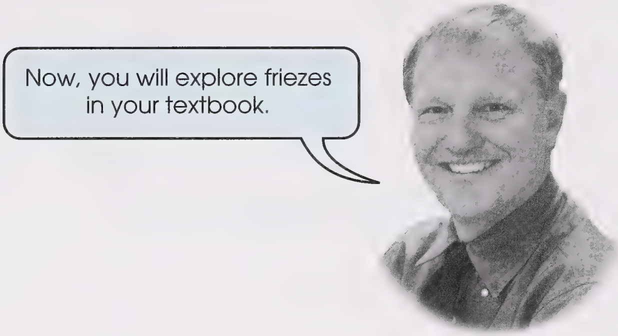
In architecture, a frieze is an ornamental band decorated with sculptures or intricate designs. The ornate frieze shown in the photograph is part of the detailing of the tomb of a fifteenth-century sultan.

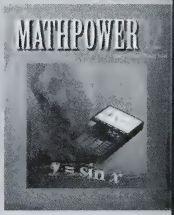


In many ancient buildings, such as temples and mosques, the designers incorporated regular patterns that can be described mathematically. As a matter of fact, in mathematics, friezes are defined as a pattern repeated in one direction. These patterns are produced using combinations of translations, reflections, and rotations.

All frieze patterns can be grouped into seven different categories. If you have access to the Internet, you can learn more about frieze patterns and their classification at the following websites:

http://www.cut-the-knot.com/triangle/Frieze.html
http://www.ucs.mun.ca/~mathed/Geometry/Transformations/frieze.html


Now, you will explore friezes in your textbook.



Turn to “CONNECTING MATH AND DESIGN” on page 61 of *MATHPOWER™ 12* and read “Frieze Patterns.”

1. Answer the following questions on pages 61 to 63 of the textbook:

   a. questions 1 to 4 of Investigation 1, “Using Transformations”
   b. questions 1 to 4 of Investigation 2, “Transforming Triangles”

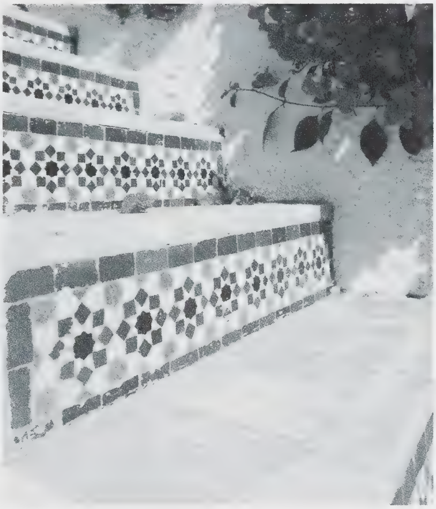
2. Where might frieze patterns appear in people’s homes?

3. Create your own frieze pattern.

Turn to page 128 to compare your responses with those in the Appendix.

Conclusion

In this section, you graphed complicated functions by applying combinations of transformations—translations, reflections, dilations, and compressions—to basic, familiar forms. You extended these procedures to analyse reciprocal functions and absolute value functions.



CORBIS/MICHAEL BOYS

The transformations you studied in this section are like the transformations many designers employ to create artistic effects in tiles, clothing, architecture, and carpets. They, too, use translations, reflections, and rotations. The photograph shows steps decorated with Moorish tiles. Which transformations can you identify in this design? Even though they may be recognizable as belonging to a particular people, the basic principles of this design, nevertheless, are universal and mathematical.

You will apply the mathematical principles you explored in this section to graph unfamiliar functions throughout Pure Mathematics 30.

Assignment

Turn to Assignment Booklet 1B and complete the assignment for Section 2.

Module Summary

In this module, you explored the transformations of functions and how these transformations are applied to graphing. These transformations included horizontal and vertical translations; expansions and compressions; and reflections across the x-axis, the y-axis, and the line $y = x$. In Section 1, you studied these transformations separately. In Section 2, you investigated the effects of combinations of transformations on the graphs of basic functions. You also analysed the graphs of reciprocal functions and absolute value functions, again working from basic forms to more-complex forms.

The transformations you explored in this module are some of the same transformations used by designers who incorporate geometric patterns in fabrics, carpets, wallpaper, tiles, flooring, and architecture. Many of these patterns also appear in traditional clothing of people from all corners of the world. The photograph shows a beaded Inuit dress. You will notice how the maker used translations, reflections, rotations and symmetry to create intricate and appealing patterns.

CORBIS/WOLFGANG KAEHLER

Final Module Assignment

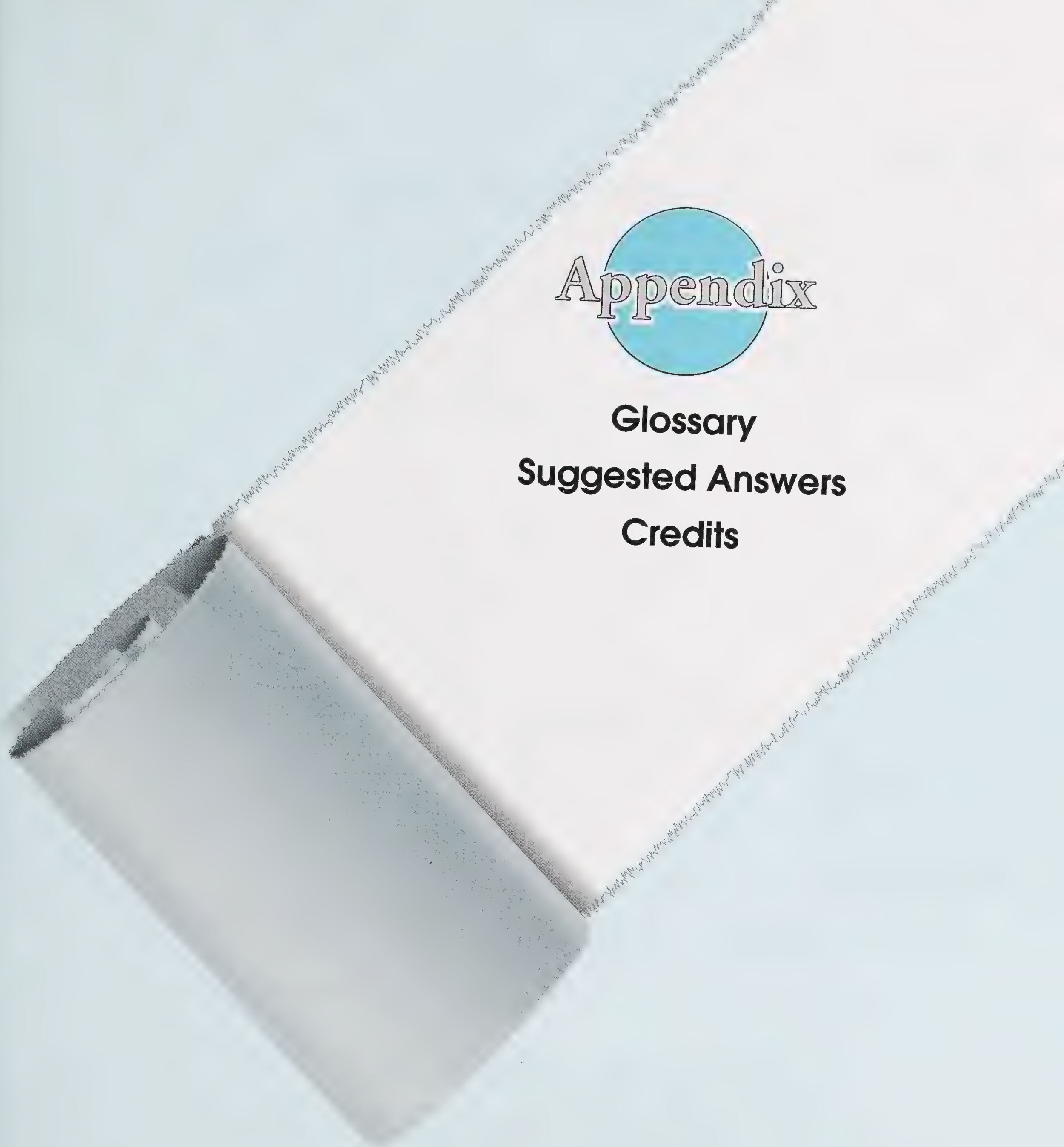
Turn to Assignment Booklet 1B and complete the final module assignment. Remember to submit your completed Assignment Booklet for assessment.

Appendix

Glossary

Suggested Answers

Credits



Glossary

absolute value function: a function of the form $y=|f(x)|$, obtained by taking the absolute value of the function $y=f(x)$

parameter: in an expression, a letter representing a constant that can be varied to produce other cases

reciprocal function: a function of the form $y=\frac{1}{f(x)}$, obtained by taking the reciprocal of the function $y=f(x)$

reflection: an image produced as if in a mirror

stretch: an expansion (dilation) or compression of a graph

translation: a transformation that produces a horizontal or vertical shift without rotation

transformation: a change in the equation and the graph of a function

Suggested Answers

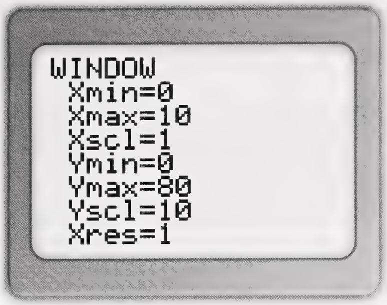
Section 1: Activity 1

1. a. **Textbook questions a. and b. of "Explore: Graph the Functions," p. 7**

a.

Bridge	Function
Ambassador Bridge	$h(t)=-5t^2+45$
Confederation Bridge	$h(t)=-5t^2+60$
Capilano Canyon Suspension Bridge	$h(t)=-5t^2+70$

b. Use the following window settings on your graphing calculator.



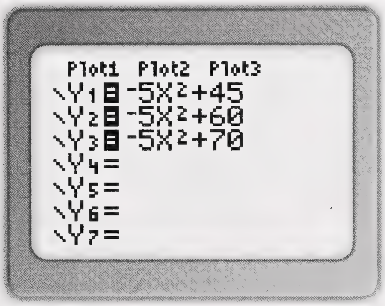
```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=80
 Yscl=10
 Xres=1
```

Now, press the following:

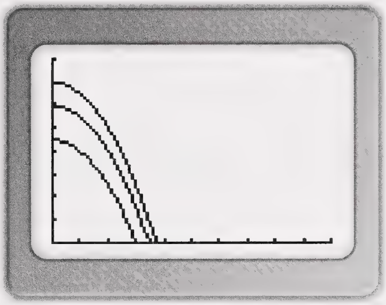
Y= (–) 5 X,T,Θ,n x^2 + 4 5 ENTER

(–) 5 X,T,Θ,n x^2 + 6 0 ENTER

(–) 5 X,T,Θ,n x^2 + 7 0 GRAPH



```
 Plot1 Plot2 Plot3
\Y1=-5X²+45
\Y2=-5X²+60
\Y3=-5X²+70
\Y4=
\Y5=
\Y6=
\Y7=
```



b. Textbook questions 1 to 5 of "Inquire," p. 7

1. The graphs appear in the first quadrant because both t and $h(t)$ are non-negative.

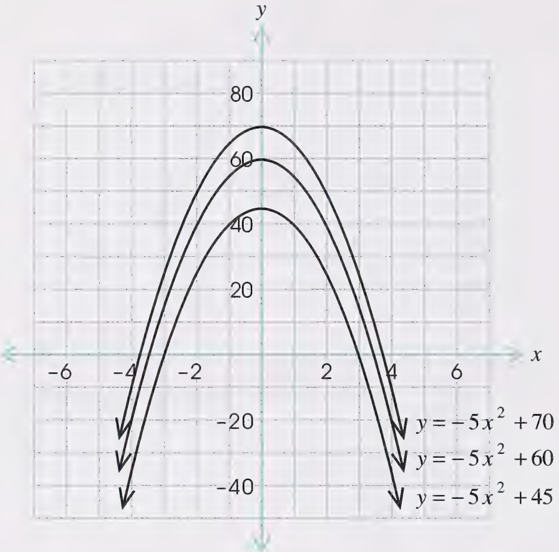
2. For any two points that have the same t-coordinate, the h-coordinate on the graph for the Confederation Bridge is 15 m larger than the h-coordinate on the graph for the Ambassador Bridge.

   Independent of the heights from which they are dropped, the distance one object falls after t seconds will be the same as the distance another object falls in the same time. Therefore, the 15-m difference between the Confederation Bridge and the Ambassador Bridge is maintained for all values of t for which both functions are defined.

Section 1: Activity 1 (continued)

3. For any two points that have the same *t*-coordinate, the *h*-coordinate on the graph for the Capilano Bridge is 25 m larger than the *h*-coordinate on the graph for the Ambassador Bridge.

   Independent of the heights from which they are dropped, the distance one object falls after *t* seconds will be the same as the distance another object falls in the same time. Therefore, the 25-m difference between the Capilano Bridge and the Ambassador Bridge is maintained for all values of *t* for which both functions are defined.

4. 

   These three graphs pass through the same points in the first quadrant as the three graphs of $h(t)$ versus *t*. However, because both *x* and *y* can be negative, the graphs also pass through points in the other three quadrants.

5. The three graphs from question 4 are narrower than the graph of $y = x^2$ over the same domain. Recall that for functions of the form $y = ax^2 + q$, the graph of the function becomes narrower as $|a|$ increases. Here, $|-5| > 1$.

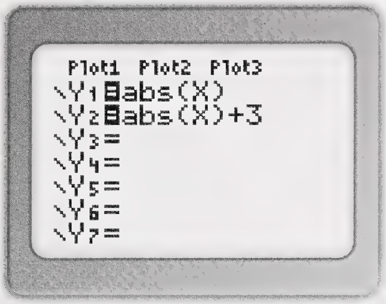
2. Textbook questions 1, 2, 21, and 22 of "Practice," pp. 10 and 11

1. The graph of $y = f(x) + 5$ is the graph of $y = f(x)$ translated 5 units up.

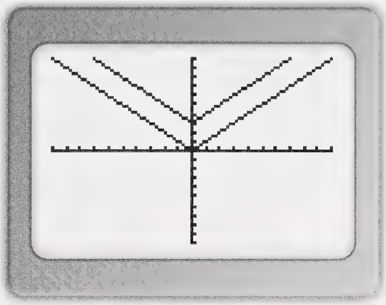
2. The graph of $y = f(x) - 6$ is the graph of $y = f(x)$ translated 6 units down.

21. The function in blue is of the form $y=f(x)+k$. Because the graph of this function is the graph of $y=|x|$ translated 3 units up, $f(x)=|x|$ and $k=3$. Therefore, the function in blue is $y=|x|+3$.

Check your answer using a graphing calculator.



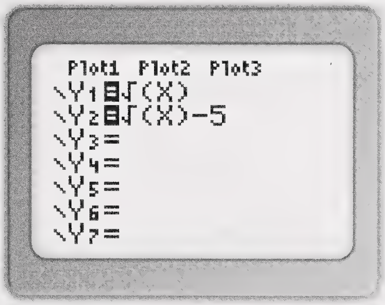
```
Plot1 Plot2 Plot3
\Y1=abs(X)
\Y2=abs(X)+3
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



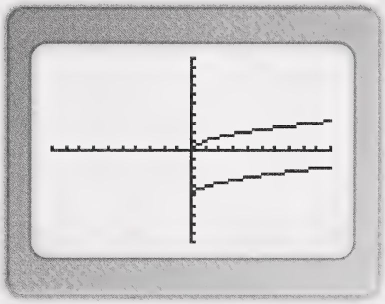
22. The function in blue is of the form $y=f(x)+k$. Because the graph of this function is the graph of $y=\sqrt{x}$ translated 5 units down, $f(x)=\sqrt{x}$ and $k=-5$. Therefore, the function in blue is

$$\begin{aligned} y &= f(x)+k \\ &= \sqrt{x}+(-5) \\ &= \sqrt{x}-5 \end{aligned}$$

Check your answer using a graphing calculator.



```
Plot1 Plot2 Plot3
\Y1=√(X)
\Y2=√(X)-5
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



3. Textbook questions 3, 4, 25, and 26 of "Practice," pp. 10 and 11

3. The graph of $y=f(x-4)$ is the graph of $y=f(x)$ translated 4 units to the right.

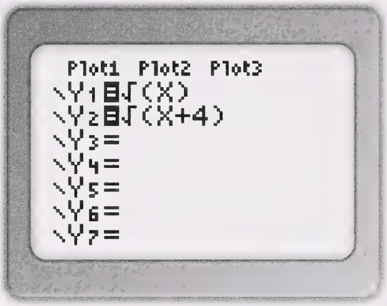
4. The graph of $y=f(x+8)$ is the graph of $y=f(x)$ translated 8 units to the left.

Section 1: Activity 1 (continued)

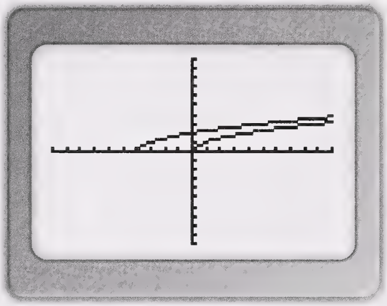
25. The function in blue is of the form $y = f(x-h)$. Because the graph of this function is the graph of $y=\sqrt{x}$ translated 4 units to the left, $f(x)=\sqrt{x}$ and $h=-4$. Therefore, the function in blue is

$$\begin{aligned} y &= f(x-h) \\ &= \sqrt{x-(-4)} \\ &= \sqrt{x+4} \end{aligned}$$

Check your answer using a graphing calculator.



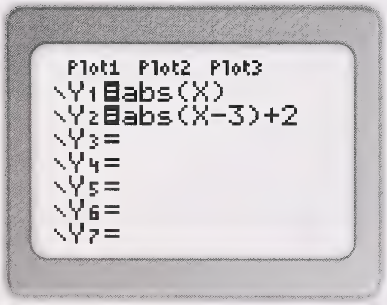
```
 Plot1 Plot2 Plot3
\Y1■√(X)
\Y2■√(X+4)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



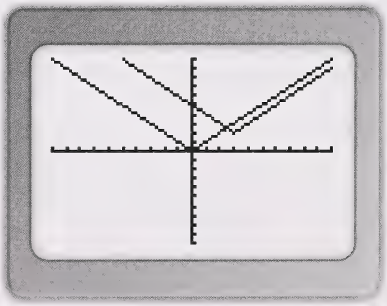
26. The function in blue is of the form $y = f(x-h)+k$. Because the graph of this function is the graph of $y=|x|$ translated 3 units to the right and 2 units up, $f(x)=|x|$, $h=3$, and $k=2$. Therefore, the function in blue is

$$\begin{aligned} y &= |x-h|+k \\ &= |x-3|+2 \end{aligned}$$

Check your answer using a graphing calculator.



```
 Plot1 Plot2 Plot3
\Y1■abs(X)
\Y2■abs(X-3)+2
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



4. **a. Textbook questions 9, 10, 14, 15, 18.f., 27, 29, 30, and 33.f. of "Practice," pp. 10 to 12**

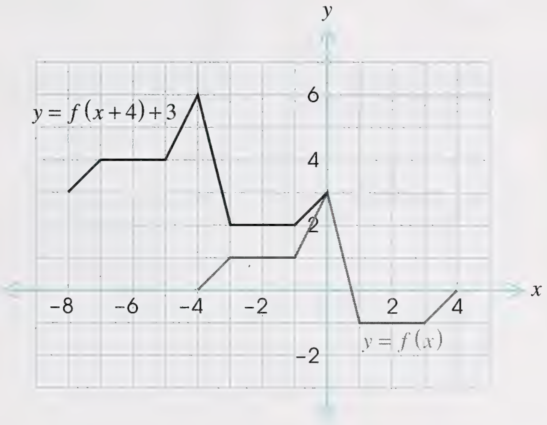
9. The graph of $y=f(x-5)-7$ is the graph of $y=f(x)$ translated 5 units to the right and 7 units down.

10. The graph of $y=f(x+2)+9$ is the graph of $y=f(x)$ translated 2 units to the left and 9 units up.

14. Because the graph of $y=f(x-h)+k$ is the graph of $y=f(x)$ translated 5 units to the left and 0 units vertically, $h=-5$ and $k=0$.

15. Because the graph of $y=f(x-h)+k$ is the graph of $y=f(x)$ translated 2 units to the left and 4 units down, $h=-2$ and $k=-4$.

18. f. The graph of $y=f(x+4)+3$ or $y=f\left(x-(-4)\right)+3$ is the graph of $y=f(x)$ translated 4 units to the left and 3 units up.



Note:

$h=-4$

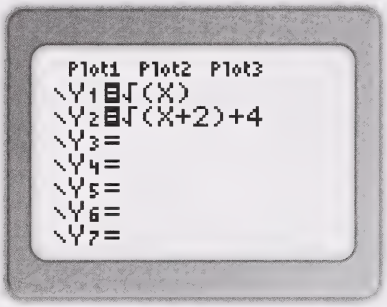
$k=3$

Section 1: Activity 1 (continued)

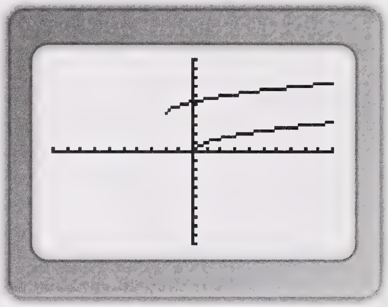
27. The graph of the function in blue is of the form $y = f(x-h)+k$. Because this graph is the graph of $y=\sqrt{x}$ translated 2 units to the left and 4 units up, $f(x)=\sqrt{x}$, $h=-2$, and $k=4$. Therefore, the function in blue is

$$\begin{aligned} y &= f(x-h)+k \\ &= \sqrt{x-(-2)}+4 \\ &= \sqrt{x+2}+4 \end{aligned}$$

Check your answer using a graphing calculator.



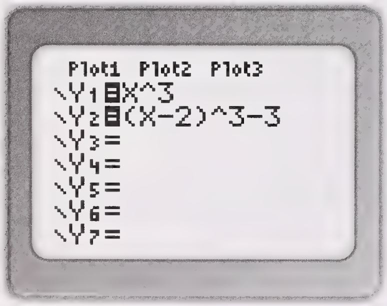
```
Plot1 Plot2 Plot3
\Y1■√(X)
\Y2■√(X+2)+4
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



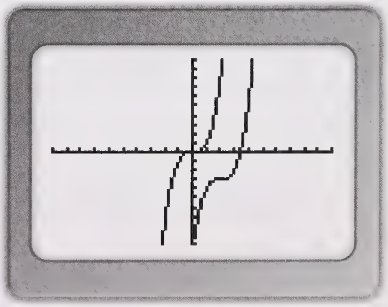
29. The graph of the function in blue is of the form $y = f(x-h)+k$. Because this graph is the graph of $y=x^3$ translated 2 units to the right and 3 units down, $f(x)=x^3$, $h=2$, and $k=-3$. Therefore, the function in blue is

$$\begin{aligned} y &= f(x-h)+k \\ &= (x-2)^3+(-3) \\ &= (x-2)^3-3 \end{aligned}$$

Check your answer using a graphing calculator.



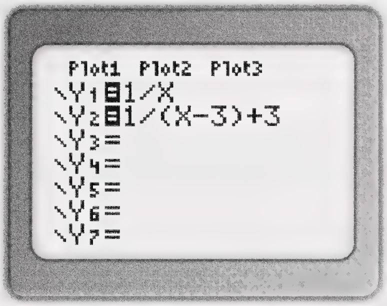
```
Plot1 Plot2 Plot3
\Y1■X^3
\Y2■(X-2)^3-3
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



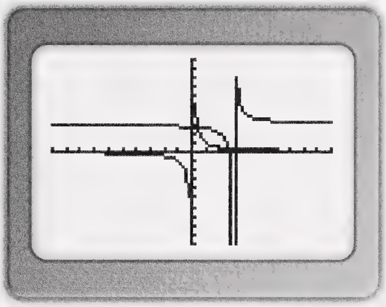
30. The graph of the function in blue is of the form $y = f(x - h) + k$. Because this graph is the graph of $y = \frac{1}{x}$ translated 3 units to the right and 3 units up, $f(x) = \frac{1}{x}$, $h = 3$, and $k = 3$. Therefore, the function in blue is

$$\begin{aligned} y &= f(x-h)+k \\ &= \frac{1}{x-3}+3 \end{aligned}$$

Check your answer using a graphing calculator.



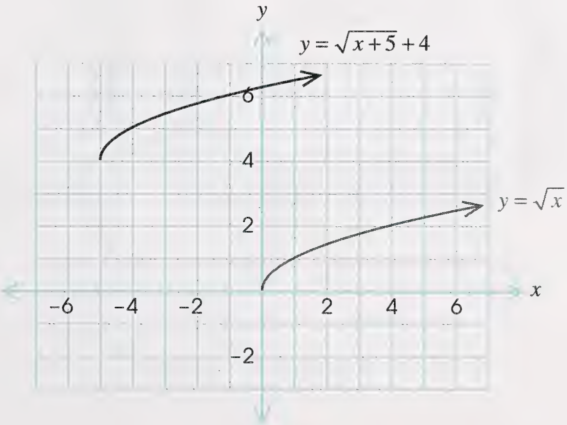
```
Plot1 Plot2 Plot3
\Y1=1/X
\Y2=1/(X-3)+3
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



33. f. Compare $y = \sqrt{x+5} + 4$ with $y = \sqrt{x-h} + k$.

$\therefore\ h = -5$ and $k = 4$

The graph of $y = \sqrt{x+5} + 4$ is the graph of $y = \sqrt{x}$ translated 5 units to the left and 4 units up.



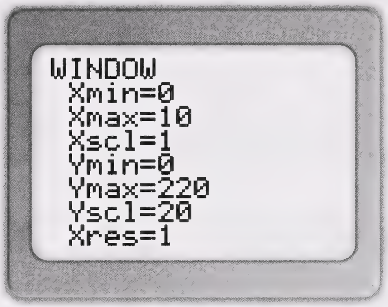
Section 1: Activity 1 (continued)

b. Textbook questions 37, 42, and 44 of "Applications and Problem Solving," pp. 12 and 13

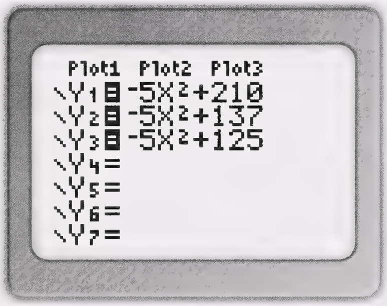
37. a.

Building	Height (m)	Function
Petro-Canada 1, Calgary	210	$h(t)=-5t^2+210$
Park Place, Vancouver	137	$h(t)=-5t^2+137$
Commerce Place, Edmonton	125	$h(t)=-5t^2+125$

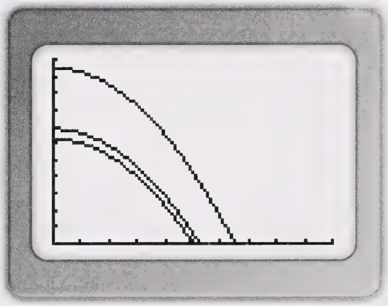
b. Graph $h(t)$ versus t in the same window of your graphing calculator.



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=220
 Yscl=20
 Xres=1
```



```
  Plot1 Plot2 Plot3
\Y1=-5X²+210
\Y2=-5X²+137
\Y3=-5X²+125
\Y4=
\Y5=
\Y6=
\Y7=
```



c. A vertical translation of 73 m downward maps the graph of $f(\text{P-C1})$ onto the graph of $f(\text{PP})$.

d. A vertical translation of 12 m upward maps the graph of $f(\text{CP})$ onto the graph of $f(\text{PP})$.

e. A vertical translation of 85 m downward maps the graph of $f(\text{P-C1})$ onto the graph of $f(\text{CP})$.

42. If the graph of $y=\sqrt{x}$ undergoes a vertical translation, the resulting function is of the form $y=\sqrt{x}+k$.

If the graph of $y=\sqrt{x}+k$ passes through the point $(4, 0)$, then

$$0=\sqrt{4}+k$$
$$0=2+k$$
$$k=-2$$

Therefore, a vertical translation of 2 units downward would result in a translation image that passes through $(4, 0)$.

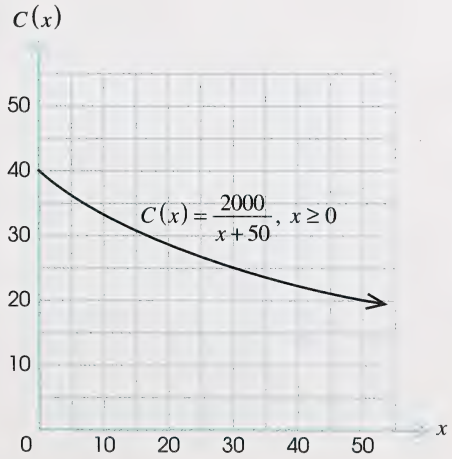
44. **a.** Let x be the volume (in mL) of water added. Therefore, the total volume (in mL) of the acid solution is $x+50$.

The volume of pure acid in the original acid solution is $50\times\frac{40}{100}=20$ mL.

The equation that describes the acid concentration, $C(x)$, as a function of the volume of water added, x, is

$$\begin{aligned}\text{Concentration} &= \frac{\text{volume of pure acid}}{\text{volume of solution}}\times 100\% \\ C(x) &= \frac{20}{x+50}\times 100 \\ &= \frac{2000}{x+50},\ x\geq 0\end{aligned}$$

b.



Section 1: Activity 1 (continued)

c.
$$\begin{aligned} C(x) &= \frac{2000}{x+50} \\ C(10) &= \frac{2000}{10+50} \\ &= \frac{2000}{60} \\ &\doteq 33.3 \end{aligned}$$

The acid concentration after 10 mL of water have been added is approximately 33.3%.

d. Let x be the volume (in mL) of water added. Therefore, the total volume (in mL) of the acid solution is $x+40$.

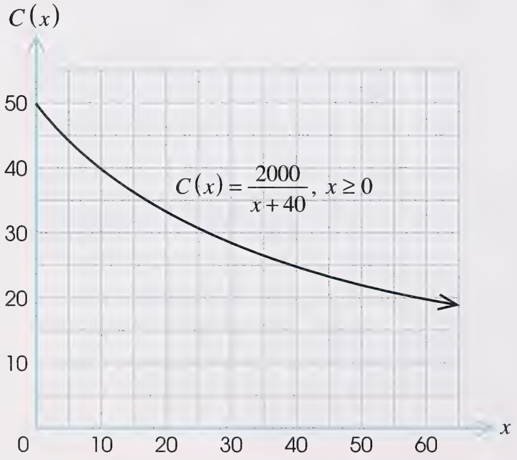
The volume of pure acid in the original acid solution is $40 \times \frac{50}{100} = 20$ mL.

The equation that describes the acid concentration, $C(x)$, as a function of the volume of water added, x, is

$$\text{Concentration} = \frac{\text{volume of pure acid}}{\text{volume of solution}} \times 100\%$$

$$\begin{aligned} C(x) &= \frac{20}{x+40} \times 100 \\ &= \frac{2000}{x+40},\ x \geq 0 \end{aligned}$$

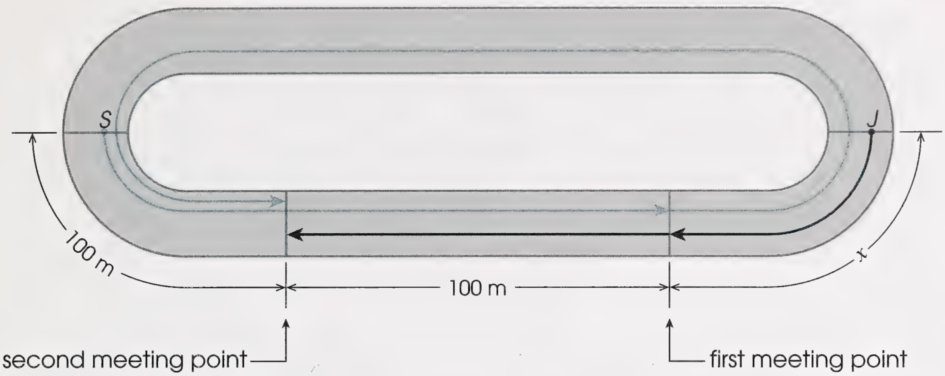
e.



f. A horizontal translation of 10 units to the left would map the graph from part e. onto the graph from part b.

5. Textbook question 2 of "Applications and Problem Solving," p. 15

2.



Let S be Samara's starting point; let J be Jennifer's starting point; and let x be the distance (in m) Jennifer ran to the first meeting point.

Because they are at opposite ends of the track, the length of the track is $2(x+200)=2x+400$ m.

When Jennifer and Samara meet at the second point, Jennifer has run a total distance of $x+100$ m and Samara has run a total distance of $2x+500$ m (the length of the track plus 100 m).

Because both Jennifer and Samara run at constant speeds, the ratios of their distances, covered in equal time periods, are equal.

Method 1

$$\begin{aligned}\frac{200}{x}&=\frac{2x+500}{x+100}\\ 200(x+100)&=(2x+500)x\\ 200x+20\ 000&=2x^2+500x\\ 0&=2x^2+300x-20\ 000\end{aligned}$$

$$\begin{aligned}2x^2+300x-20\ 000&=0\\ x^2+150x-10\ 000&=0\\ (x+200)(x-50)&=0\end{aligned}$$

$$\therefore\ x+200=0 \quad \text{or} \quad x-50=0$$
$$x=-200 \qquad\qquad x=50$$

Method 2

$$\begin{aligned}\frac{x}{x+100}&=\frac{200}{2x+500}\\ x(2x+500)&=200(x+100)\\ 2x^2+500x&=200x+20\ 000\\ 2x^2+300x-20\ 000&=0\\ x^2+150x-10\ 000&=0\\ (x+200)(x-50)&=0\end{aligned}$$

Section 1: Activity 1 (continued)

Because length is non-negative, $x = 50$.

$$\begin{aligned} \therefore\ 2x + 400 &= 2(50) + 400 \\ &= 100 + 400 \\ &= 500 \end{aligned}$$

The length of the track is 500 m.

Section 1: Activity 2

1. a. **Textbook question "Explore: Graph the Functions," p. 16**

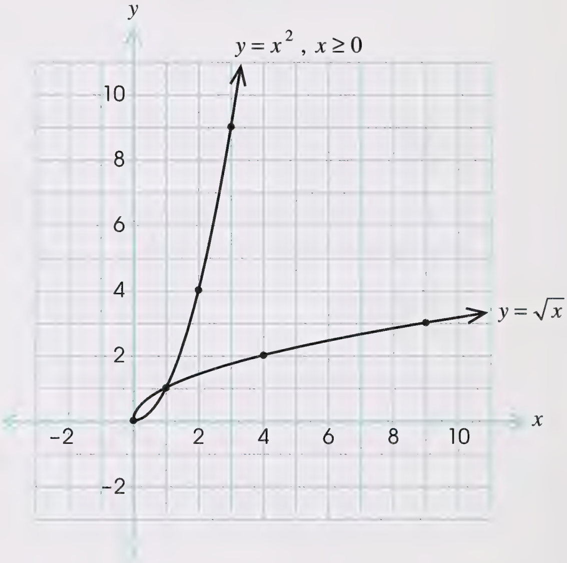
$y = x^2,\ x \geq 0$

x	y
0	0
1	1
2	4
3	9
4	16

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4

Note: The restriction, $x \geq 0$, has been added to $y = x^2$ because x cannot be negative.



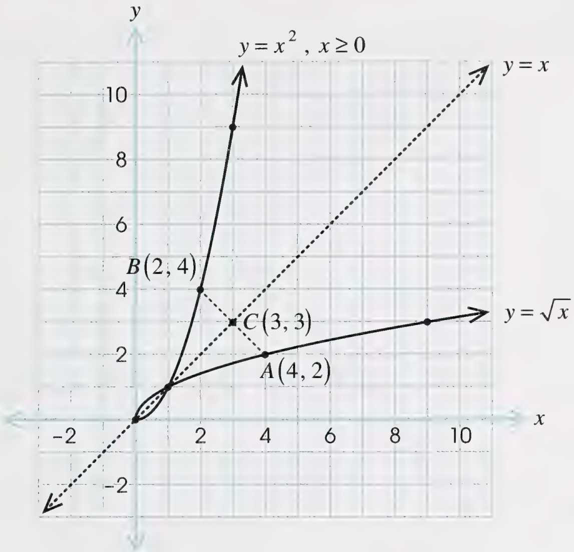
b. **Textbook questions 1 to 7 of "Inquire," p. 16**

1. The graphs are drawn in the first quadrant because area, A, and side length, s, are non-negative.

2. If the coordinates of the five points in the table for $y = x^2$, where $x \geq 0$, are reversed, the resulting points lie on the graph of $y = \sqrt{x}$.

3. If the coordinates of the five points in the table for $y = \sqrt{x}$ are reversed, the resulting points lie on the graph of $y = x^2$, where $x \geq 0$.

4.



5. Answers may vary. A sample answer is given.

Join points $A(4, 2)$ and $B(2, 4)$; then determine the distances A and B lie from the line $y = x$. Notice that, from the graph, segment AB passes through point $C(3, 3)$ and that the slope of this segment is -1.

Now, the slope of $y = x$ is $+1$. Therefore, segment AB is perpendicular to $y = x$. As a result, the line segment joining $A(4, 2)$ and $C(3, 3)$ is perpendicular to $y = x$.

The distance from $A(4, 2)$ to $y = x$ is the length of the line segment joining $A(4, 2)$ and $C(3, 3)$.

Section 1: Activity 2 (continued)

Let $(x_1, y_1) = (4, 2)$ and $(x_2, y_2) = (3, 3)$.

$$\begin{aligned}\therefore AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-4)^2 + (3-2)^2} \\ &= \sqrt{(-1)^2 + 1^2} \\ &= \sqrt{1+1} \\ &= \sqrt{2}\end{aligned}$$

Similarly, the line segment joining $B(2, 4)$ and $C(3, 3)$ is perpendicular to $y = x$.

The distance from $B(2, 4)$ to $y = x$ is the length of the line segment joining $B(2, 4)$ and $C(3, 3)$.

Let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (3, 3)$.

$$\begin{aligned}\therefore BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-2)^2 + (3-4)^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{1+1} \\ &= \sqrt{2}\end{aligned}$$

Therefore, the distances $A(4, 2)$ and $B(2, 4)$ are the same distance from the line $y = x$.

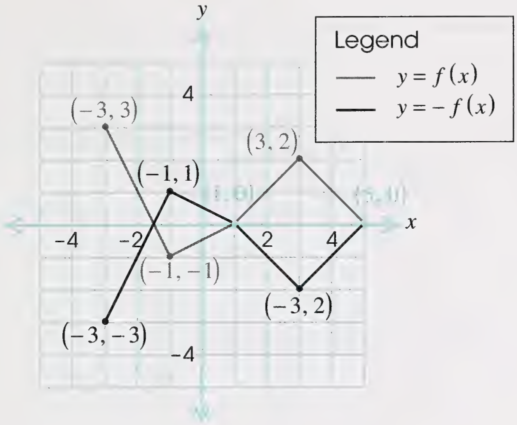
6. The line $y = x$ is the line of reflection. This line acts as a mirror reflecting the graph of $y = x^2$, where $x \geq 0$, onto the graph of $y = \sqrt{x}$, and vice versa.

7.
$$\begin{aligned}s &= \sqrt{A} \\ &= \sqrt{35\ 300} \\ &\doteq 188\end{aligned}$$

The length of the base of the Bent Pyramid is about 188 m.

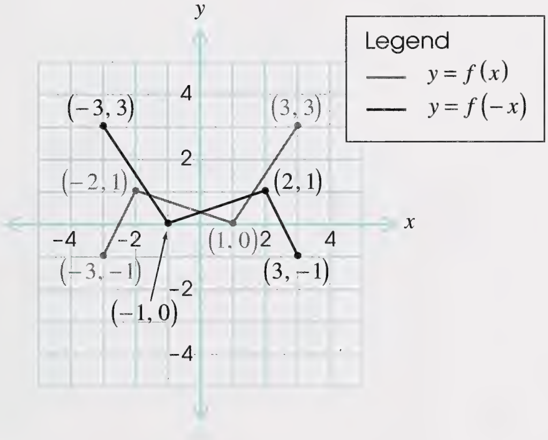
2. a. Textbook questions 2, 3, 7, 8, 13, and 14 of “Practice,” pp. 25 and 26

2.



Note: The coloured dots on the graph show the common points on the graphs of $y = f(x)$ and $y = -f(x)$.

3.



7. Substitute $-x$ for *x* in $y = x - 3$ to reflect its graph in the *y*-axis. Therefore, the equation of the blue graph is $y = -x - 3$.

8. Substitute $-x$ for *x* in $y = \sqrt{x + 2}$ to reflect its graph in the *y*-axis. Therefore, the equation of the blue graph is $y = \sqrt{-x + 2}$ or $y = \sqrt{2 - x}$.

Section 1: Activity 2 (continued)

13. Substitute $-|x-4|$ for $|x-4|$ to reflect the graph of $y=|x-4|$ in the x-axis. Therefore, the equation of the blue graph is $y=-|x-4|$.

14. Substitute $-\frac{4}{x^2+1}$ for $\frac{4}{x^2+1}$ to reflect the graph of $y=\frac{4}{x^2+1}$ in the x-axis. Therefore, the equation of the blue graph is

$$y=-\frac{4}{x^2+1}$$

b. Textbook questions 32 and 34 of "Applications and Problem Solving," p. 27

32. The altitudes of the two triangular faces are mirror images in the y-axis.

Substitute $-x$ for x in the given altitude.

$$\begin{aligned} y &= 1.27x+146 \\ &= 1.27(-x)+146 \\ &= -1.27x+146 \end{aligned}$$

The equation of the other altitude is $y=-1.27x+146$.

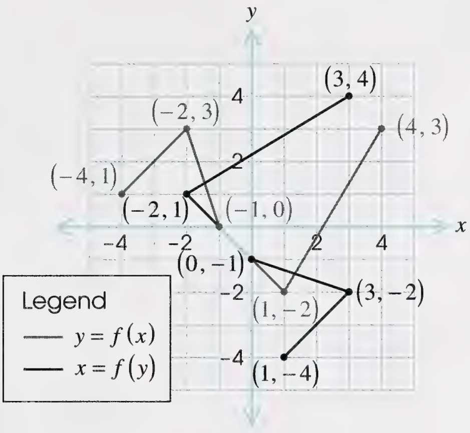
34. $$\begin{aligned} y &= x^2-2x \\ &= -\left(-x^2+2x\right) \\ &= -\left(2x-x^2\right) \end{aligned}$$

Therefore, the graphs of $y=x^2-2x$ and $y=2x-x^2$ are mirror images in the x-axis.

Remember: *The graph of $y=-f(x)$ is the graph of $y=f(x)$ reflected in the x-axis.*

3. a. Textbook questions 5, 15, 16, 17, 18, and 26 of "Practice," pp. 25 to 27

5. Interchange the coordinates of the points on $y = f(x)$ to obtain $x = f(y)$.



Note: *The colour portion of the graph shows where the graphs of $y = f(x)$ and $x = f(y)$ overlap.*

15. The graph of $x = f(y)$ is the graph of $y = f(x)$ reflected in the line $y = x$.

Interchange x and y in the equation $y = 2x + 3$; then solve for y in terms of x.

$$\begin{aligned} y &= 2x - 3 \\ x &= 2y + 3 \quad \leftarrow \text{Interchange } x \text{ and } y. \\ 2y + 3 &= x \\ 2y &= x - 3 \\ y &= \frac{x-3}{2} \end{aligned}$$

The equation of the blue graph is $y = \frac{x-3}{2}$.

Section 1: Activity 2 (continued)

16. The graph of $x = f(y)$ is the graph of $y = f(x)$ reflected in the line $y = x$.

Interchange x and y in the equation $y = x^2 + 2$; then solve for y in terms of x.

$$\begin{aligned} y &= x^2 + 2 \\ x &= y^2 + 2 \quad \leftarrow \text{[illegible]} \\ y^2 + 2 &= x \\ y^2 &= x - 2 \\ y &= \pm\sqrt{x-2} \end{aligned}$$

The equation of the blue graph is $y = \pm\sqrt{x-2}$.

17. The graph of $x = f(y)$ is the graph of $y = f(x)$ reflected in the line $y = x$.

Interchange x and y in the equation $y = x^3$; then solve for y in terms of x.

$$\begin{aligned} y &= x^3 \\ x &= y^3 \quad \leftarrow \text{[illegible]} \\ y^3 &= x \\ y &= \sqrt[3]{x} \end{aligned}$$

Therefore, the equation of the blue graph is $y = \sqrt[3]{x}$.

18. The graph of $x = f(y)$ is the graph of $y = f(x)$ reflected in the line $y = x$.

Interchange x and y in the equation $y = \sqrt{x-4}$; then solve for y in terms of x.

$$\begin{aligned} y &= \sqrt{x-4} \\ x &= \sqrt{y-4} \quad \leftarrow \text{[illegible]} \\ \sqrt{y-4} &= x \\ \left(\sqrt{y-4}\right)^2 &= x^2,\ x \geq 0 \\ y - 4 &= x^2,\ x \geq 0 \\ y &= x^2 + 4,\ x \geq 0 \end{aligned}$$

The restriction on x is necessary because x is a non-negative root in the equation $x = \sqrt{y-4}$.

Therefore, the equation of the blue graph is $y = x^2 + 4$, where $x \geq 0$.

26. **a.**

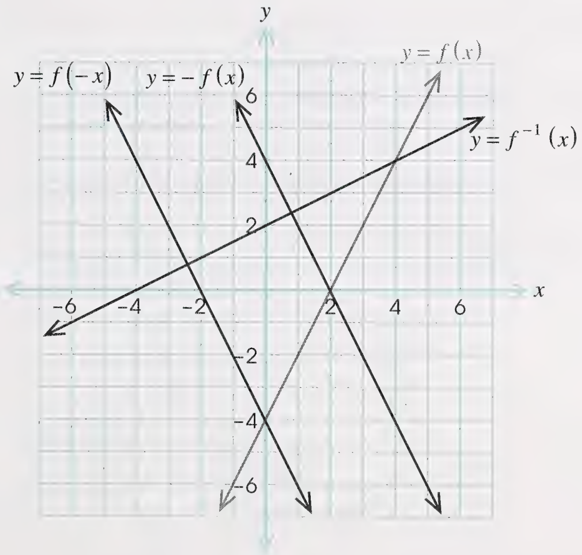
$$\begin{aligned} y &= -f(x) \\ &= -(2x-4) \\ &= -2x+4 \end{aligned} \qquad\qquad \begin{aligned} y &= f(-x) \\ &= 2(-x)-4 \\ &= -2x-4 \end{aligned}$$

To find $y = f^{-1}(x)$, interchange x and y in the equation $y = 2x-4$ and solve for y in terms of x.

$$\begin{aligned} y &= 2x-4 \\ x &= 2y-4 \quad \longleftarrow \text{Interchange } x \text{ and } y. \\ 2y-4 &= x \\ 2y &= x+4 \\ y &= \frac{x+4}{2} \\ &= \frac{1}{2}x+2 \end{aligned}$$

$$\therefore f^{-1}(x) = \frac{1}{2}x+2$$

b.



Section 1: Activity 2 (continued)

c. The point that is invariant for the reflection in the *x*-axis is the *x*-intercept of $y=2x-4$.

Let $y=0$.

$$\begin{aligned} \therefore\ y&=2x-4 \\ 0&=2x-4 \\ 4&=2x \\ x&=2 \end{aligned}$$

Therefore, the point $(2,\ 0)$ is invariant for the reflection in the *x*-axis.

The point that is invariant for the reflection in the *y*-axis is the *y*-intercept of $y=2x-4$.

Let $x=0$.

$$\begin{aligned} \therefore\ y&=2x+4 \\ &=2(0)-4 \\ &=-4 \end{aligned}$$

Therefore, the point $(0,\ -4)$ is invariant for the reflection in the *y*-axis.

The point that is invariant for the reflection in the line $y=x$ is the point of intersection of $y=2x-4$ and $y=x$.

$\therefore\ y=2x-4$ ① $y=x$ ②

Substitute *x* for *y* into ①.

$$\begin{aligned} y&=2x-4 \\ x&=2x-4 \\ -x&=-4 \\ x&=4 \end{aligned}$$

Substitute $x = 4$ into ①.

$$\begin{aligned} y &= 2x - 4 \\ &= 2(4) - 4 \\ &= 4 \end{aligned}$$

The point of intersection is $(4, 4)$.

Therefore, the point $(4, 4)$ is invariant for the reflection in the line $y = x$.

b. Textbook questions 33 and 36 of "Applications and Problem Solving," p. 27

33. The graphs of $y = f(x)$ and $x = f(y)$ are mirror images in the line $y = x$. Therefore, substitute *y* for *x* in the equation $x = 2$.

The line obtained by reflecting the line $x = 2$ in the line $y = x$ is $y = 2$.

36. a. To find the *y*-intercept, let $x = 0$.

$$\begin{aligned} \therefore f(x) &= x^2 - x - 6 \\ f(0) &= 0^2 - 0 - 6 \\ &= -6 \end{aligned}$$

The *y*-intercept is -6.

Therefore, $f(x) = x^2 - x - 6$ crosses the *y*-axis at $(0, -6)$.

To find the *x*-intercepts of $f(x) = x^2 - x - 6$, let $f(x) = 0$.

$$\begin{aligned} \therefore f(x) &= x^2 - x - 6 \\ 0 &= x^2 - x - 6 \\ 0 &= (x - 3)(x + 2) \end{aligned}$$

$$\begin{aligned} \therefore x - 3 &= 0 \quad \text{or} \quad & x + 2 &= 0 \\ x &= 3 & x &= -2 \end{aligned}$$

The *x*-intercepts are 3 and -2.

Therefore, $f(x) = x^2 - x - 6$ crosses the *x*-axis at $(3, 0)$ and $(-2, 0)$.

Section 1: Activity 2 (continued)

b. The following partial tables of values can be used to draw the graphs of $y=f(x)$, $y=-f(x)$, $y=f(-x)$, and $x=f(y)$.

$y=f(x)$

x	y
-2	0
0	-6
3	0

$y=-f(x)$

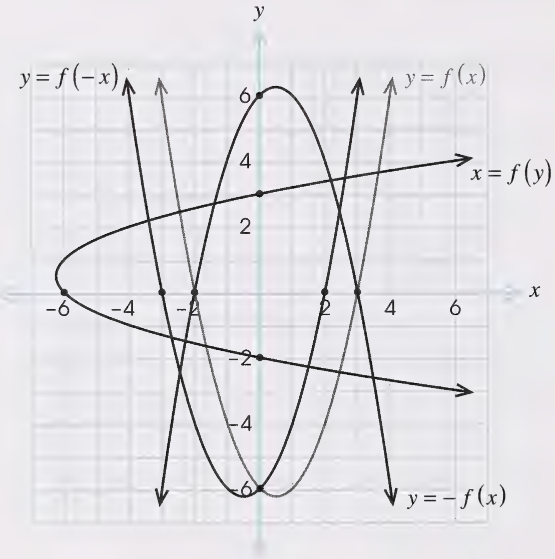
x	y
-2	0
0	6
3	0

$y=f(-x)$

x	y
2	0
0	-6
-3	0

$x=f(y)$

x	y
0	-2
-6	0
0	3



4. a. Textbook questions 25, 28, 29, and 31 of "Practice," p. 27

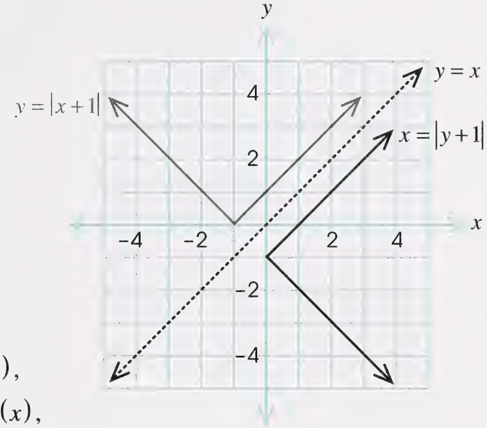
25. To reflect $f(x)=|x+1|$ in the line $y=x$, interchange the coordinates of the ordered pairs that satisfy $y=f(x)$.

$y=|x+1|$

x	-3	-2	-1	0	1
y	2	1	0	1	2

$x=|y+1|$

x	2	1	0	1	2
y	-3	-2	-1	0	1



For each element, except 0, in the domain of $k(x)$, there are two elements in the range. Therefore, $k(x)$, the reflection of $f(x)=|x+1|$ in the line $y=x$, is not a function.

28. a. $y=-f(x)$
$=-(x^3-3)$
$=-x^3+3$

$y=f(-x)$
$=(-x)^3-3$
$=-x^3-3$

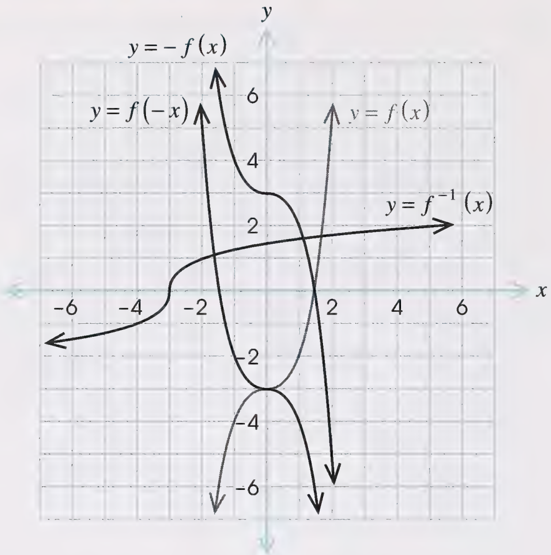
To obtain $f^{-1}(x)$, interchange x and y in $y=f(x)$ and solve for y in terms of x.

$$y=x^3-3$$
$$x=y^3-3$$ ← Interchange x and y
$$y^3-3=x$$
$$y^3=x+3$$
$$y=\sqrt[3]{x+3}$$

$\therefore f^{-1}(x)=\sqrt[3]{x+3}$

Section 1: Activity 2 (continued)

b.



29. **a.** $y = -f(x)$ $\qquad\qquad$ $y = f(-x)$

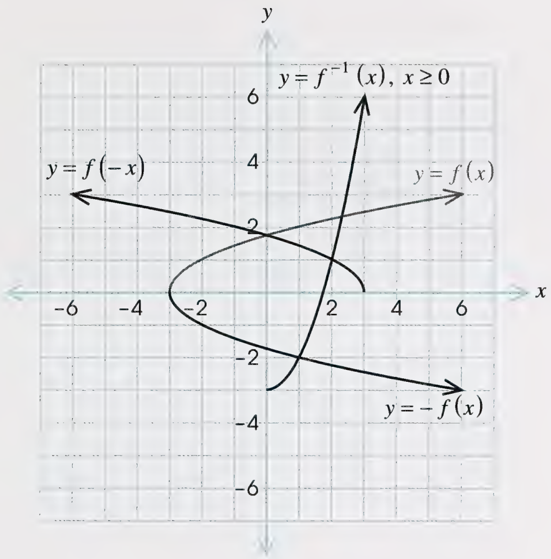
$= -\sqrt{x+3}$ $\qquad\qquad$ $= \sqrt{-x+3}$

To obtain $f^{-1}(x)$, interchange x and y in $y = \sqrt{x+3}$ and solve for y in terms of x.

$$
\begin{aligned}
y &= \sqrt{x+3} \\
x &= \sqrt{y+3} \quad \longleftarrow \text{interchange } x \text{ and } y \\
x^2 &= y+3,\ x \geq 0 \\
y+3 &= x^2,\ x \geq 0 \\
y &= x^2 - 3,\ x \geq 0
\end{aligned}
$$

$\therefore f^{-1}(x) = x^2 - 3,\ x \geq 0$

b.



31. **a.** $y = -f(x)$

$$= -\frac{6}{x^2+2}$$

$y = f(-x)$

$$= \frac{6}{(-x)^2+2}$$

$$= \frac{6}{x^2+2}$$

To obtain $f^{-1}(x)$, interchange x and y in $y = \frac{6}{x^2+2}$ and solve for y in terms of x.

$$y = \frac{6}{x^2+2}$$

$$x = \frac{6}{y^2+2} \quad \longleftarrow \text{Interchange } x \text{ and } y.$$

$$x(y^2+2) = 6$$

$$y^2+2 = \frac{6}{x}$$

$$y^2 = \frac{6}{x} - 2$$

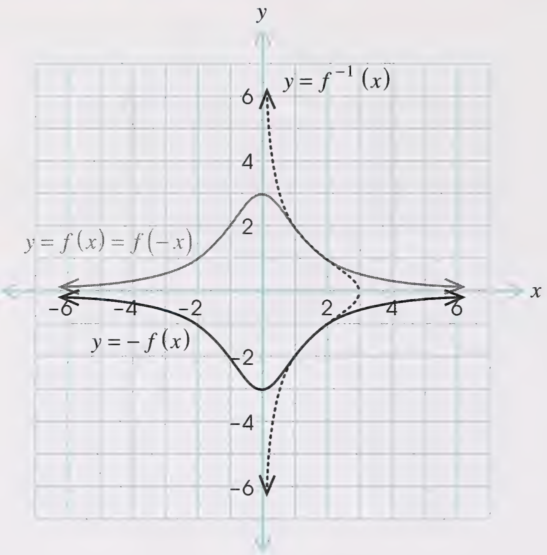
$$y^2 = \frac{6-2x}{x}$$

$$y = \pm\sqrt{\frac{6-2x}{x}}$$

$$\therefore f^{-1}(x) = \pm\sqrt{\frac{6-2x}{x}}$$

Section 1: Activity 2 (continued)

b.



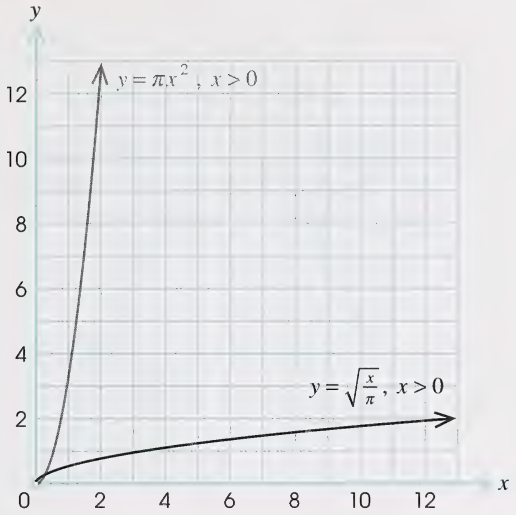
c. For $f(x) = \frac{6}{x^2+2}$ and $f(-x) = \frac{6}{x^2+2}$, their graphs are symmetrical about the *y*-axis. Because $x^2 \geq 0$, *y* is positive for all real values of *x*. The graphs of $y = f(x)$ and $y = f(-x)$ have a maximum value of 3 when $x = 0$. Also, as $|x|$ increases, the value of *y* approaches 0. Thus, the *x*-axis is a horizontal asymptote of these graphs.

For $y = -f(x)$, the graph is symmetrical about the *y*-axis. Because $x^2 \geq 0$, *y* is negative for all real values of *x*. It has a minimum value of -3 when $x = 0$. Also, as $|x|$ increases, the value of *y* approaches 0. Thus, the *x*-axis is a horizontal asymptote of this graph.

For $y = f^{-1}(x)$, the graph is symmetrical about the *x*-axis. Because $y^2 \geq 0$, *x* is positive for all real values of *y*. It has a maximum *x*-value of 3 when $y = 0$. Also, as $|y|$ increases, the value of *x* approaches 0. Thus, the *y*-axis is a vertical asymptote of this graph.

b. Textbook questions 39, 42, and 45 of “Applications and Problem Solving,” p. 28

39. a.



b. $A=\pi r^2,\ r>0$

$\pi r^2=A,\ r>0$

$r^2=\dfrac{A}{\pi},\ r>0$

$r=\sqrt{\dfrac{A}{\pi}}$

This function models finding the radius of a circle given the area of the circle.

c. Refer to the answer to question 39.a.

d. The two functions are inverses.

42. Original function

$y=f(x)$

x	y
0	1
1	4
2	0
3	−2
5	0

Reflection in y-axis

$y=f(-x)$

x	y
0	1
−1	4
−2	0
−3	−2
−5	0

Reflection in x-axis

$y=-f(-x)$

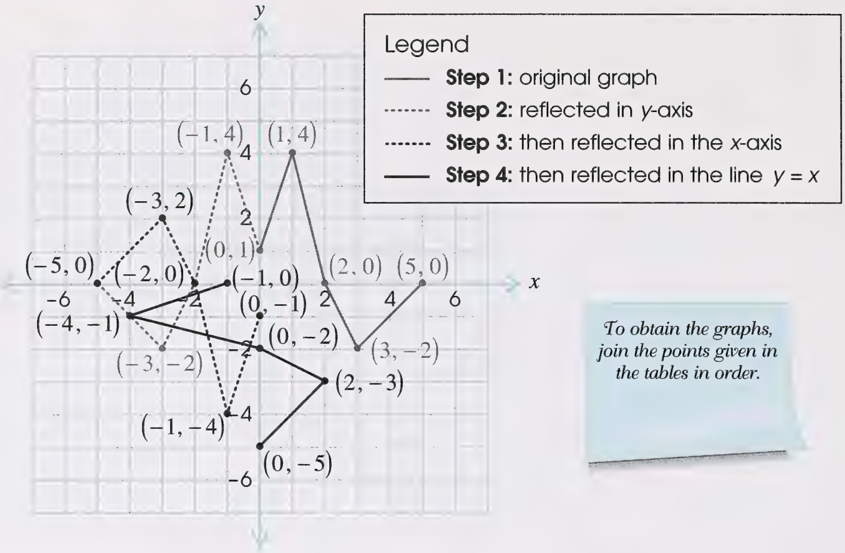
x	y
0	−1
−1	−4
−2	0
−3	2
−5	0

Inverse

$x=-f(-y)$

x	y
−1	0
−4	−1
0	−2
2	−3
0	−5

Section 1: Activity 2 (continued)



To obtain the graphs, join the points given in the tables in order.

45. The graph of $y=f(-x)$ is the graph of $y=f(x)$ reflected in the *y*-axis. The point that is invariant for the reflection in the *y*-axis is the *y*-intercept of $y=x^2-3$.

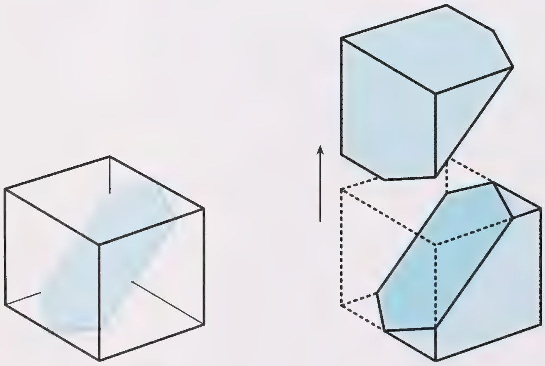
Let $x=0$.

$$\begin{aligned} \therefore\ y &= 0^2-3 \\ &= -3 \end{aligned}$$

Therefore, the point $(0,-3)$ is invariant for this transformation.

5. Textbook question 5 of "Applications and Problem Solving," p. 15

5.



Section 1: Activity 3

1. a. **Textbook question "Explore: Graph the Functions," p. 30**

$y=\sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4

$y=2\sqrt{x}$

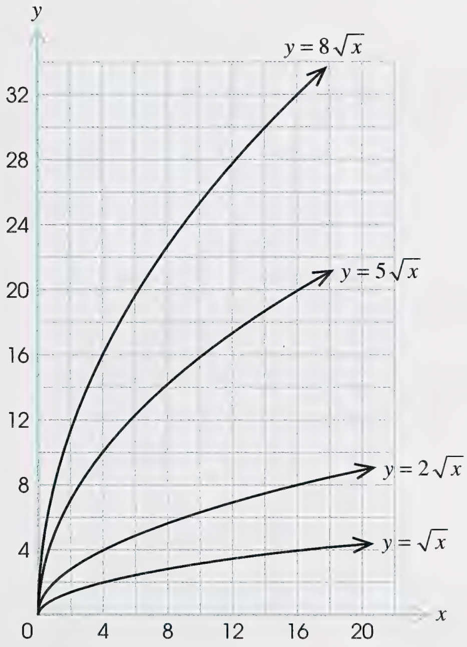
x	y
0	0
1	2
4	4
9	6
16	8

$y=5\sqrt{x}$

x	y
0	0
1	5
4	10
9	15
16	20

$y=8\sqrt{x}$

x	y
0	0
1	8
4	16
9	24
16	32



b. **Textbook questions 1 to 4 of "Inquire," p. 30**

1. A point on the graph of $y=2\sqrt{x}$ has a y-coordinate that is two times larger than the y-coordinate of the point on the graph of $y=\sqrt{x}$ with the same non-zero x-coordinate.

Section 1: Activity 3 (continued)

2. A point on the graph of $y = 5\sqrt{x}$ has a y-coordinate that is 5 times larger than the y-coordinate of the point on the graph of $y = \sqrt{x}$ with the same non-zero x-coordinate.

3. A point on the graph of $y = 8\sqrt{x}$ has a y-coordinate that is 8 times larger than the y-coordinate of the point on the graph of $y = \sqrt{x}$ with the same non-zero x-coordinate.

4. **a.**

$$\begin{aligned} T &= 8\sqrt{\ell} \\ 9.8 &= 8\sqrt{\ell} \\ \sqrt{\ell} &= \frac{9.8}{8} \\ \ell &= \left(\frac{9.8}{8}\right)^2 \\ &\doteq 1.5 \end{aligned}$$

The length of pendulum is approximately 1.5 m.

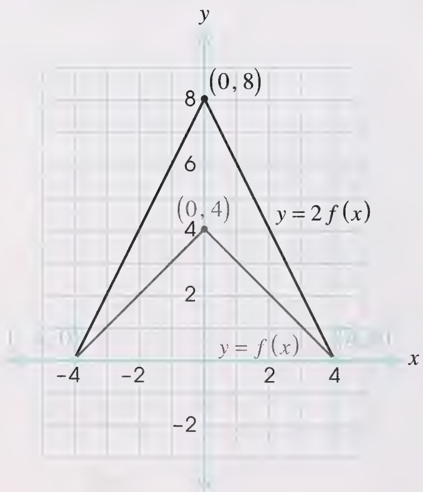
b.

$$\begin{aligned} T &= 5\sqrt{\ell} \\ &\doteq 5\sqrt{1.5} \\ &\doteq 6.1 \end{aligned}$$

The period of Little Ben's pendulum would be approximately 6.1 s.

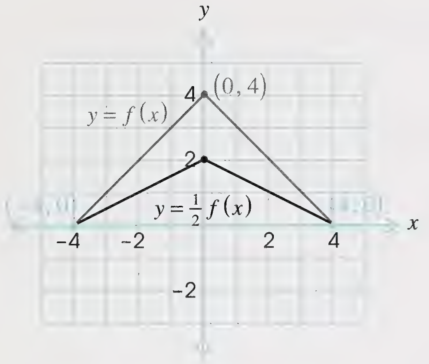
2. a. Textbook questions 1, 2, 7, 13, 15, 16, 20, 21, 33, 34, 35, and 38 of "Practice," pp. 38 and 39

1.



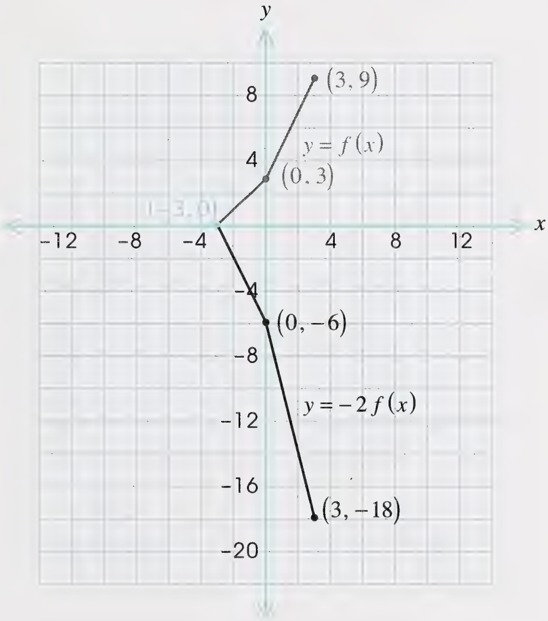
Note: The colour portions of the graph show where the graphs of $y = f(x)$ and $y = 2f(x)$ overlap.

2.



Note: *The colour portions of the graph show where the graphs of $y = f(x)$ and $y = \frac{1}{2} f(x)$ overlap.*

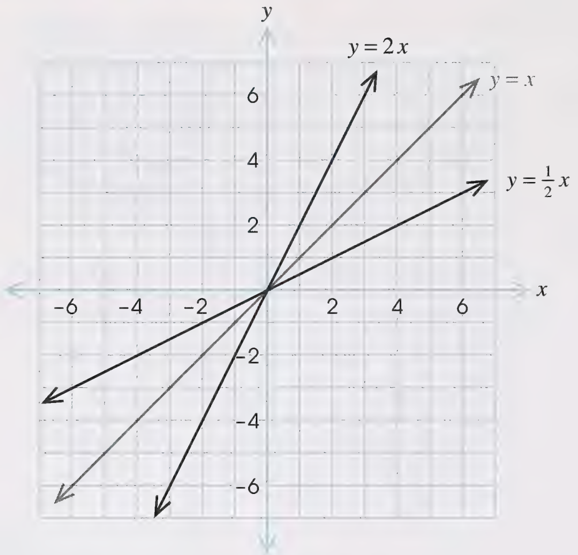
7.



Note: *The colour portions of the graph show where the graphs of $y = f(x)$ and $y = -2f(x)$ overlap.*

Section 1: Activity 3 (continued)

13. **a.**

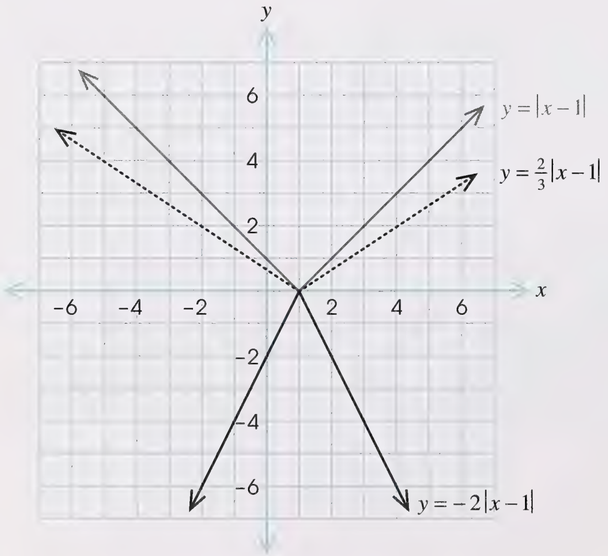


b. The graph of $y = 2x$ is the graph of $y = x$ expanded vertically by a factor of 2.

The graph of $y = \frac{1}{2}x$ is the graph of $y = x$ compressed vertically by a factor of $\frac{1}{2}$.

c. The origin, $(0, 0)$, is an invariant point.

15. **a.**

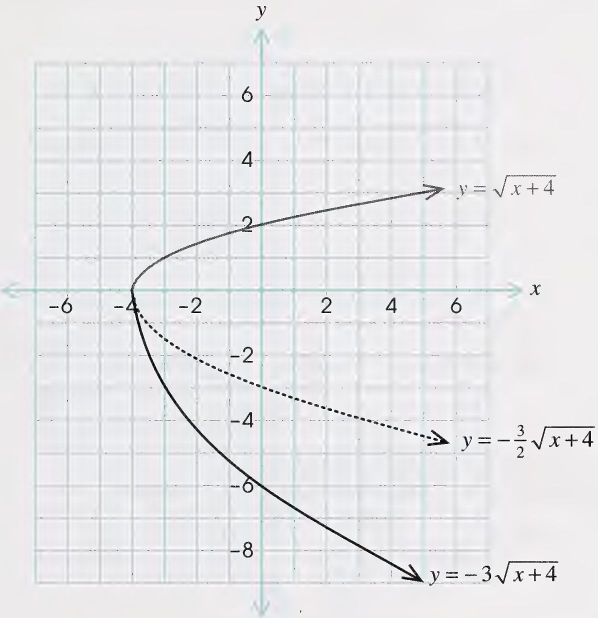


b. The graph of $y=-2|x-1|$ is the graph of $y=|x-1|$ expanded vertically by a factor of 2 and reflected in the x-axis.

The graph of $y=\frac{2}{3}|x-1|$ is the graph of $y=|x-1|$ compressed vertically by a factor of $\frac{2}{3}$.

c. The point $(1, 0)$ is an invariant point.

16. **a.**



b. The graph of $y=-3\sqrt{x+4}$ is the graph of $y=\sqrt{x+4}$ expanded vertically by a factor of 3 and reflected in the x-axis.

The graph of $y=-\frac{3}{2}\sqrt{x+4}$ is the graph of $y=\sqrt{x+4}$ expanded vertically by a factor of $\frac{3}{2}$ and reflected in the x-axis.

c. The point $(-4, 0)$ is an invariant point.

20. The graph of $y=\frac{1}{2}f(x)$ is the graph of $y=f(x)$ compressed vertically by a factor of $\frac{1}{2}$.

21. The graph of $y=-2f(x)$ is the graph of $y=f(x)$ expanded vertically by a factor of 2 and reflected in the x-axis.

Section 1: Activity 3 (continued)

33. The blue graph is the graph of $y=|x|$ expanded vertically by a factor of 2. Therefore, the equation of the blue graph is $y=2|x|$.

34. The blue graph is the graph of $y=x^2$ compressed vertically by a factor of $\frac{1}{2}$. Therefore, the equation of the blue graph is $y=\frac{1}{2}x^2$.

35. The blue graph is the graph of $y=x+1$ expanded vertically by a factor of 2. Therefore, the equation of the blue graph is $y=2(x+1)$ or $y=2x+2$.

38. The blue graph is the graph of $y=\frac{4}{x^2+1}$ expanded vertically by a factor of 2. Therefore, the equation of the blue graph is $y=2\left(\frac{4}{x^2+1}\right)$ or $y=\frac{8}{x^2+1}$.

b. Textbook questions 43, 45, 50.c., and 51.a. of "Applications and Problem Solving," pp. 39 and 40

43. a. For dry asphalt,

$$\begin{aligned} d(s)&=0.006s^2 \\ d(80)&=0.006(80)^2 \\ &=38.4 \end{aligned}$$

For wet asphalt,

$$\begin{aligned} d(s)&=0.009s^2 \\ d(80)&=0.009(80)^2 \\ &=57.6 \end{aligned}$$

For black ice,

$$\begin{aligned} d(s)&=0.04s^2 \\ d(80)&=0.04(80)^2 \\ &=256 \end{aligned}$$

The stopping distance for a car travelling 80 km/h is approximately 38.4 m on dry asphalt, 57.6 m on wet asphalt, and 256 m on black ice.

b. Answers may vary.

Reasonable speeds are from 0 km/h to 120 km/h; so, the domain of functions for stopping distance is $0 \le s \le 120$.

For each function, the range is $d(0) \le d(s) \le d(120)$

For dry ashphalt,

$$0 \le d(s) \le 0.006(120)^2$$
$$0 \le d(s) \le 86 \text{ m}$$

For wet ashphalt,

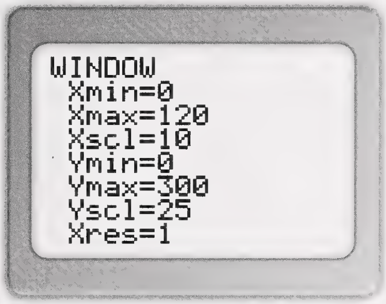
$$0 \le d(s) \le 0.009(120)^2$$
$$0 \le d(s) \le 129 \text{ m}$$

For black ice,

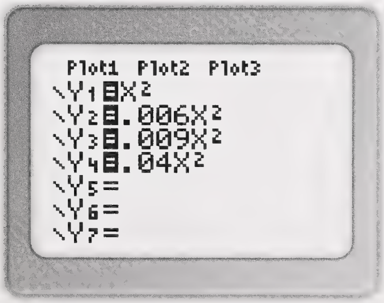
$$0 \le d(s) \le 0.04(120)^2$$
$$0 \le d(s) \le 576 \text{ m}$$

On black ice, it is unlikely you would skid 576 m without hitting something or going into the ditch.

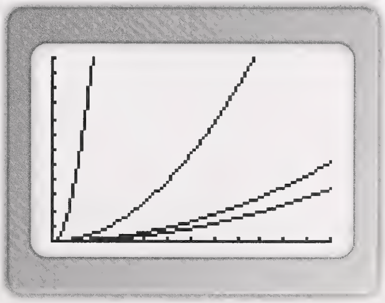
c. Graph the functions on your graphing calculator.



```
WINDOW
 Xmin=0
 Xmax=120
 Xscl=10
 Ymin=0
 Ymax=300
 Yscl=25
 Xres=1
```



```
 Plot1 Plot2 Plot3
\Y1=X²
\Y2=.006X²
\Y3=.009X²
\Y4=.04X²
\Y5=
\Y6=
\Y7=
```



Section 1: Activity 3 (continued)

d. The graph of $y = 0.006x^2$ is the graph of $y = x^2$ compressed vertically by a factor of 0.006.

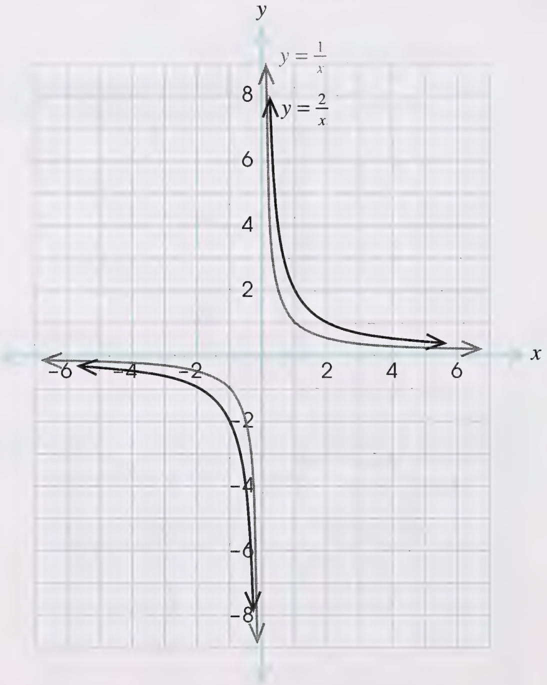
The graph of $y = 0.009x^2$ is the graph of $y = x^2$ compressed vertically by a factor of 0.009.

The graph of $y = 0.04x^2$ is the graph of $y = x^2$ compressed vertically by a factor of 0.04.

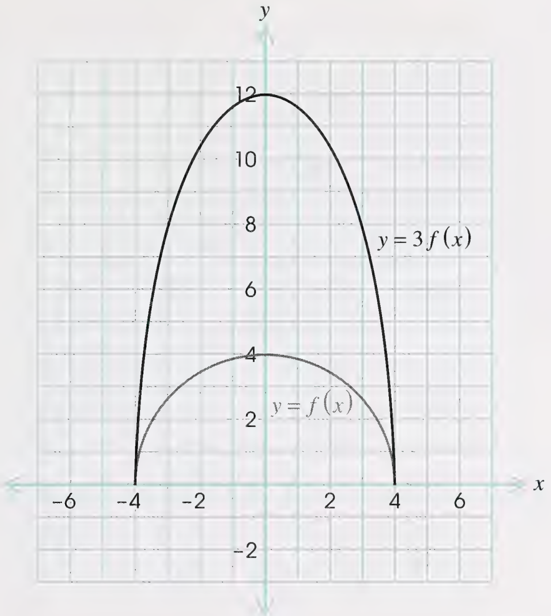
45. When the graph of $y = f(x)$ is expanded vertically by a factor of 3 and reflected in the *x*-axis, it becomes $y = -3f(x)$.

Compare $y = -3f(x)$ with $y = af(kx)$. Therefore, $a = -3$ and $k = 1$.

50. **c.** The graph of $y = \frac{2}{x}$ is the graph of $y = \frac{1}{x}$ expanded vertically by a factor of 2. If you are given the graph of $y = \frac{1}{x}$, simply plot points twice as far from the *x*-axis to sketch $y = \frac{2}{x}$.

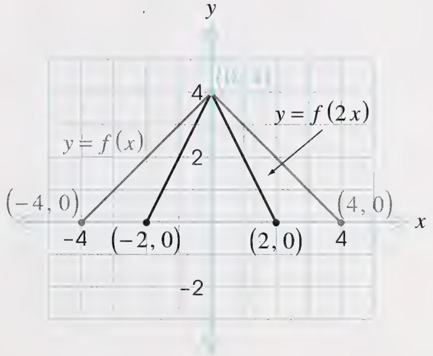


51. a. Multiply the y-coordinates on the given graph by 3 to obtain the graph of $y = 3f(x)$.



3. a. Textbook questions 3, 4, 9, 10, 18, 24, 25, and 41.b. of "Practice," pp. 38 and 39

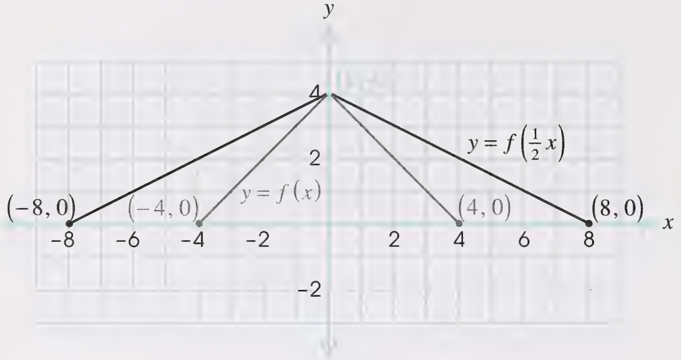
3. The graph of $y = f(2x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of $\frac{1}{2}$.



Note: The colour portion of the graph shows where the graphs of $y = f(x)$ and $y = f(2x)$ overlap.

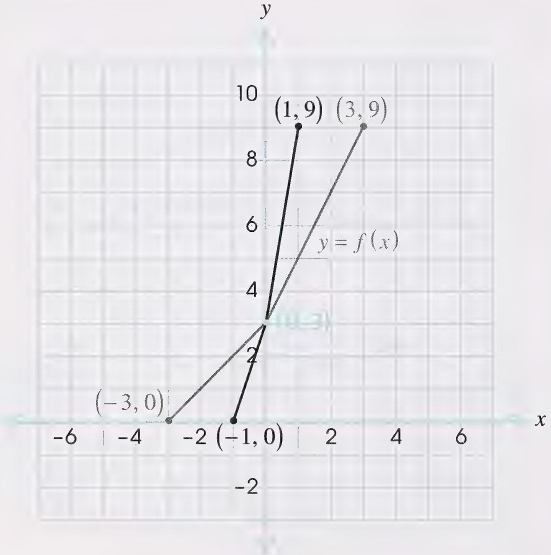
Section 1: Activity 3 (continued)

4. The graph of $y=f\left(\frac{1}{2}x\right)$ is the graph of $y=f(x)$ expanded horizontally by a factor of 2.



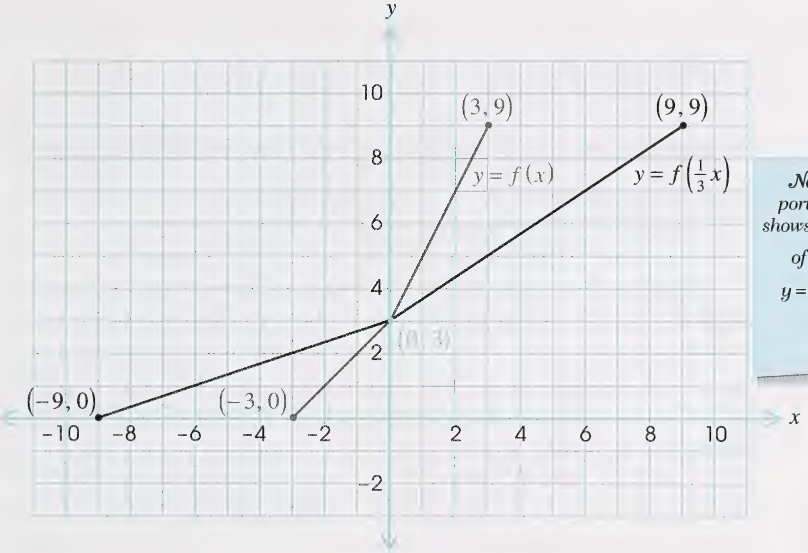
***Note:** The colour portion of the graph shows where the graphs of $y=f(x)$ and $y=f\left(\frac{1}{2}x\right)$ overlap.*

9. The graph of $y=f(3x)$ is the graph of $y=f(x)$ compressed horizontally by a factor of $\frac{1}{3}$.



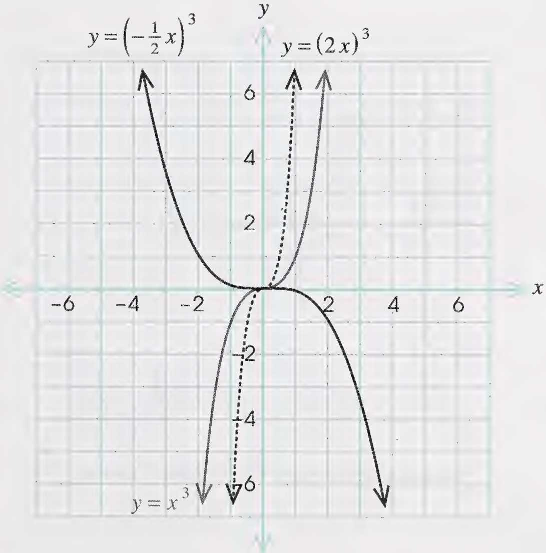
***Note:** The colour portion of the graph shows where the graphs of $y=f(x)$ and $y=f(3x)$ overlap.*

10. The graph of $y = f\left(\frac{1}{3}x\right)$ is the graph of $y = f(x)$ expanded horizontally by a factor of 3.



Note: *The colour portion of the graph shows where the graphs of* $y = f(x)$ *and* $y = f\left(\frac{1}{3}x\right)$ *overlap.*

18. a.



Section 1: Activity 3 (continued)

b. The graph of $y=\left(-\frac{1}{2}x\right)^3$ is the graph of $y=x^3$ expanded horizontally by a factor of 2 and reflected in the *y*-axis.

The graph of $y=(2x)^3$ is the graph of $y=x^3$ compressed horizontally by a factor of $\frac{1}{2}$.

c. The origin, $(0, 0)$, is an invariant point.

24. The graph of $y=f\left(\frac{1}{2}x\right)$ is the graph of $y=f(x)$ expanded horizontally by a factor of 2.

25. The graph of $y=f(-4x)$ is the graph of $y=f(x)$ compressed horizontally by a factor of $\frac{1}{4}$ and reflected in the *y*-axis.

41. b. The zeros of $f(x)=(x+4)(x-2)$ are -4 and 2. Recall that the zeros of the function are the *x*-intercepts of its graph. Therefore, the graph of $y=f(x)$ crosses the *x*-axis at $(-4, 0)$ and $(2, 0)$.

The graph of $y=f\left(\frac{1}{2}x\right)$ is the graph of $y=f(x)$ expanded horizontally by a factor of 2.

Therefore, the graph of $y=f\left(\frac{1}{2}x\right)$ crosses the *x*-axis at

$$\begin{aligned}\left(\frac{x}{k}, y\right)&=\left(\frac{-4}{\frac{1}{2}}, 0\right) \quad \text{and} \quad \left(\frac{x}{k}, y\right)=\left(\frac{2}{\frac{1}{2}}, 0\right)\\ &=(-8, 0) \qquad\qquad\qquad\quad =(4, 0)\end{aligned}$$

The zeros of the function $y=f\left(\frac{1}{2}x\right)$ are -8 and 4.

b. Textbook questions 46, 47, and 51.c. of "Applications and Problem Solving," p. 40

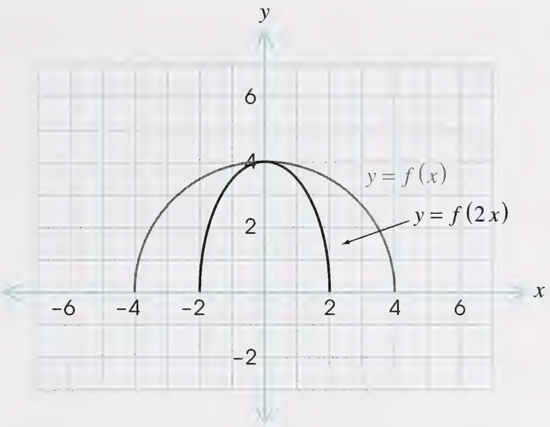
46. When the graph of $y=f(x)$ is compressed horizontally by a factor of $\frac{1}{3}$, it becomes $y=f(3x)$.

Compare $y=f(3x)$ with $y=af(kx)$. Therefore, $a=1$ and $k=3$.

47. When the graph of $y=f(x)$ is expanded horizontally by a factor of 2 and reflected in the *y*-axis, it becomes $y=f\left(-\frac{1}{2}x\right)$.

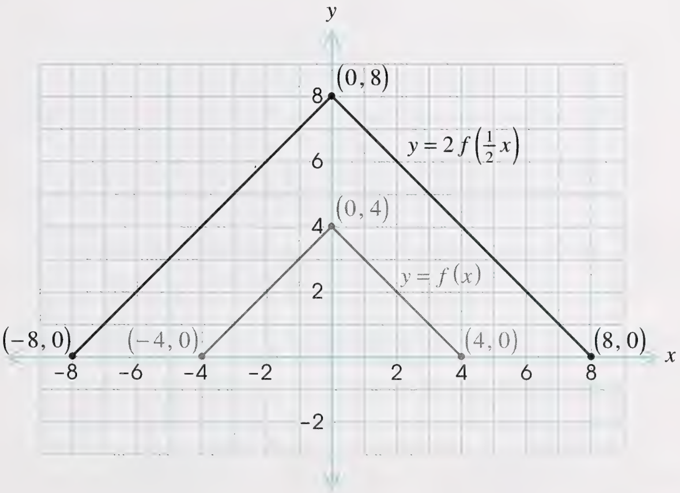
Compare $y=f\left(-\frac{1}{2}x\right)$ with $y=af(kx)$. Therefore, $a=1$ and $k=-\frac{1}{2}$.

51. **c.** Divide the *x*-coordinates on the given graph by 2 to obtain the graph of $y = f(2x)$.



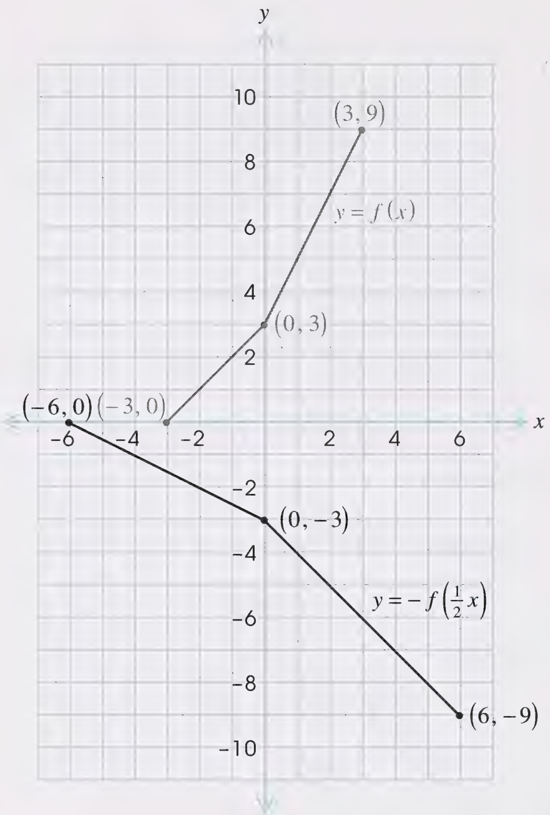
4. **a.** **Textbook questions 5, 11, 27, 29, 32, and 41.e. of "Practice," pp. 38 and 39**

5. The graph of $y = 2f\left(\frac{1}{2}x\right)$ is the graph of $y = f(x)$ expanded vertically by a factor of 2 and horizontally by a factor of 2.



Section 1: Activity 3 (continued)

11. The graph of $y = -f\left(\frac{1}{2}x\right)$ is the graph of $y = f(x)$ expanded horizontally by a factor of 2 and reflected in the x-axis.



27. The graph of $y = 3f(2x)$ is the graph of $y = f(x)$ expanded vertically by a factor of 3 and compressed horizontally by a factor of $\frac{1}{2}$.

29. The graph of $y = 4f\left(\frac{1}{2}x\right)$ is the graph of $y = f(x)$ expanded vertically by a factor of 4 and horizontally by a factor of 2.

32. The graph of $y = 5f\left(-\frac{1}{2}x\right)$ is the graph of $y = f(x)$ expanded vertically by a factor of 5 and horizontally by a factor of 2 and reflected in the y-axis.

41. **e.** The zeros of $f(x)=(x+4)(x-2)$ are -4 and 2.

The graph of $y=f(2x)$ is the graph of $y=f(x)$ compressed horizontally by a factor of $\frac{1}{2}$.

Therefore, the zeros of $y=f(2x)$ are

$$\left(\frac{x}{k},0\right)=\left(\frac{-4}{2},0\right) \text{ and } \left(\frac{x}{k},y\right)=\left(\frac{2}{2},0\right)$$
$$=(-2,0) \qquad\qquad =(1,0)$$

b. Textbook questions 48 and 49 of "Applications and Problem Solving," p. 40

48. When the graph $y=f(x)$ is compressed vertically by a factor of of $\frac{1}{2}$, it becomes $y=\frac{1}{2}f(x)$.

When the graph of $y=\frac{1}{2}f(x)$ is expanded horizontally by a factor of 3, it becomes $y=\frac{1}{2}f\left(\frac{1}{3}x\right)$.

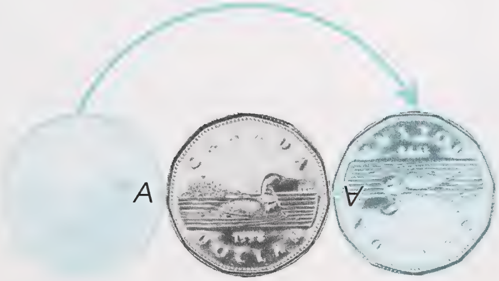
Compare $y=\frac{1}{2}f\left(\frac{1}{3}x\right)$ with $y=af(kx)$. Therefore, $a=\frac{1}{2}$ and $k=\frac{1}{3}$.

49. When the graph of $y=f(x)$ is expanded vertically by a factor of 2, it becomes $y=2f(x)$.

When the graph of $y=2f(x)$ is compressed horizontally by a factor of $\frac{1}{4}$, it becomes $y=2f(4x)$.

Compare $y=2f(4x)$ with $y=af(kx)$. Therefore, $a=2$ and $k=4$.

5. Slide the first loonie half way around the circumference of the second loonie, keeping point A on the first loonie in contact with the second loonie. The first coin will make one half turn, appearing upside down on the right.



Section 1: Activity 3 (continued)

Next, carefully roll the first loonie around the second loonie through the same distance. Now, because the first loonie will rotate through half its circumference, it will make a second half turn. The first coin will now be right side up!

Section 1: Follow-up Activities

Extra Help

1. First, draw the graph of $y = x^2$.

x	0	±1	±2
y	0	1	4

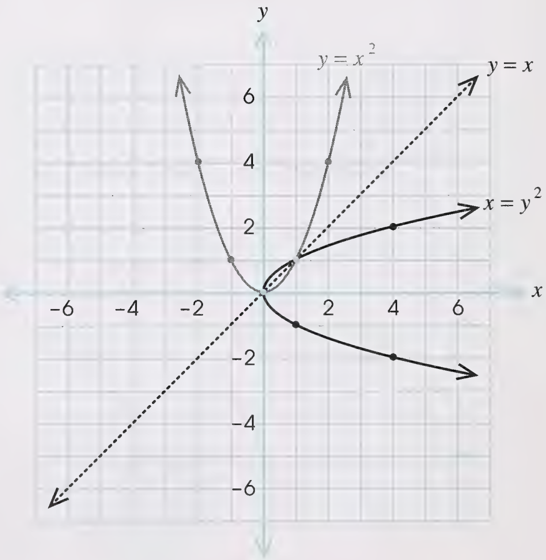
Next, reflect the graph of $y = x^2$ in the line $y = x$ to obtain the graph of $x = y^2$. Select points from $y = x^2$, and plot their inverses.

$y = x^2$

x	y
0	0
±1	1
±2	4

$x = y^2$

x	y
0	0
1	±1
4	±2



The graph of $x = y^2$ should look like the one shown on the right.

2. First, draw the graph of $y=|x|$.

x	0	±1	±2	±3	±4
y	0	1	2	3	4

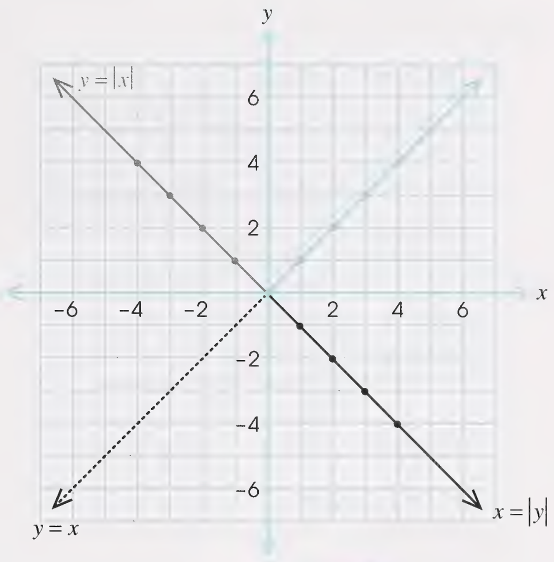
Next, reflect the graph of $y=|x|$ in the line $y=x$ to obtain the graph of $x=|y|$.

$y=|x|$

x	y
0	0
±1	1
±2	2
±3	3
±4	4

$x=|y|$

x	y
0	0
1	±1
2	±2
3	±3
4	±4



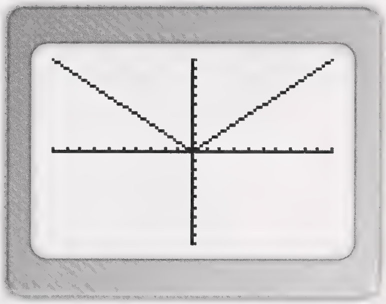
Note: The colour portion of the graph shows where the graphs of $y=|x|$, $x=|y|$, and $y=x$ overlap.

The graph of $x=|y|$ should look like the one shown on the right.

Enrichment

1. a. Substitute $-x$ for x.

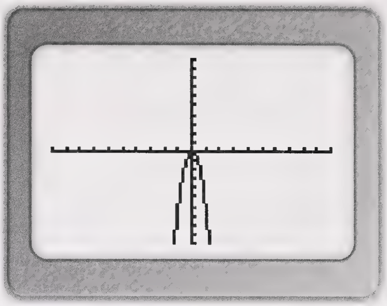
$$\begin{aligned} f(x) &= |x| \\ f(-x) &= |-x| \\ &= |x| \\ &= f(x) \end{aligned}$$



Because $f(-x)=f(x)$, the function $f(x)=|x|$ or $y=|x|$ is symmetric with respect to the y-axis.

Enrichment (continued)

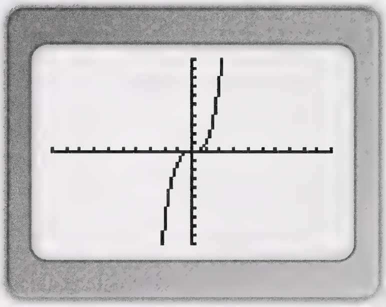
b. Substitute $-x$ for x.



$$\begin{aligned} f(x) &= -x^4 - 5x^2 \\ f(-x) &= -(-x)^4 - 5(-x)^2 \\ &= -x^4 - 5x^2 \\ &= f(x) \end{aligned}$$

Because $f(-x) = f(x)$, the function $f(x) = -x^4 - 5x^2$ is symmetric with respect to the y-axis.

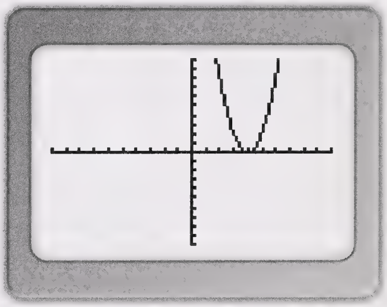
c. Substitute $-x$ for x.



$$\begin{aligned} f(x) &= x^3 \\ f(-x) &= (-x)^3 \\ &= -x^3 \\ &\neq f(x) \end{aligned}$$

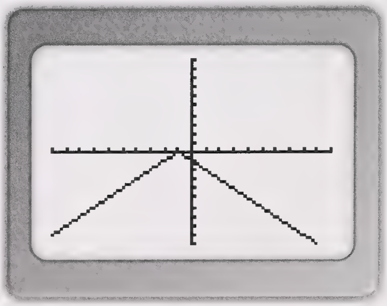
Because $f(-x) \neq f(x)$, the function $f(x) = x^3$ is not symmetric with respect to the y-axis.

2. a. The axis of symmetry of $f(x) = 2x^2$ lies along the y-axis. The graph of the function $f(x) = 2(x-4)^2$ is the graph of $f(x) = 2x^2$ translated 4 units to the right. Therefore, the axis of symmetry is also moved 4 units to the right.



The equation of the axis of symmetry is $x = 4$.

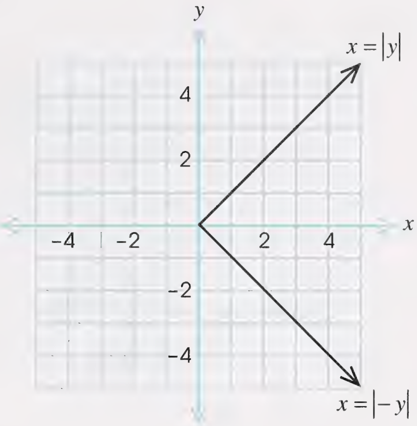
b. The axis of symmetry of $y = -|x|$ lies along the y-axis. The graph of $y = -|x+1|$ is the graph of $y = -|x|$ translated 1 unit to the left. Therefore, the axis of symmetry of $y = -|x+1|$ is $x = -1$.



3. **a.** Substitute $-y$ for y.

$$\begin{aligned} x &= |y| \\ &= |-y| \\ &= |y| \end{aligned}$$

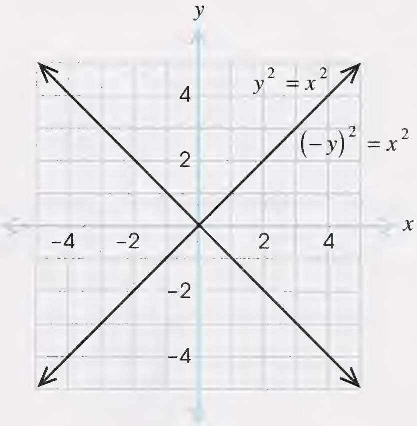
Because the result is identical to the original relation, $x = |y|$ is symmetric with respect to the x-axis.



b. Substitute $-y$ for y.

$$\begin{aligned} y^2 &= x^2 \\ (-y)^2 &= x^2 \\ y^2 &= x^2 \end{aligned}$$

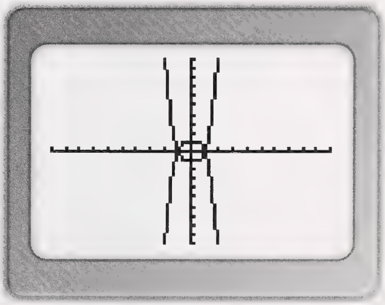
Because the result is identical to the original relation, $y^2 = x^2$ is symmetric with respect to the x-axis.



c. Substitute $-y$ for y.

$$\begin{aligned} y &= x^4 - 1 \\ (-y) &= x^4 - 1 \\ y &= -x^4 + 1 \end{aligned}$$

Because the result is not the same as the original relation, $y = x^4 - 1$ is not symmetric with respect to the x-axis.

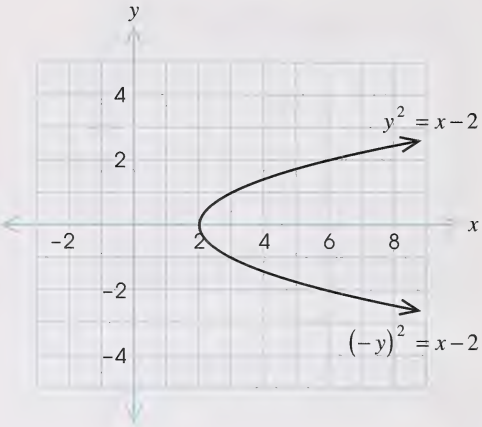


Enrichment (continued)

d. Substitute $-y$ for y.

$$y^2 = x-2$$
$$(-y)^2 = x-2$$
$$y^2 = x-2$$

Because the result is the same as the original relation, $y^2 = x-2$ is symmetric with respect to the x-axis.

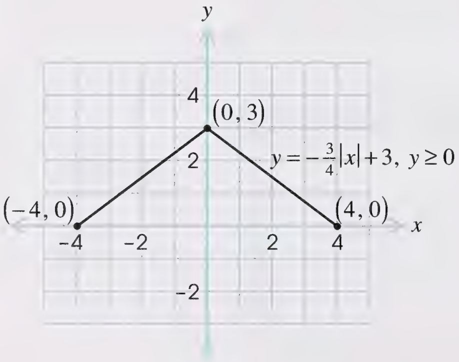


Section 2: Activity 1

1. **a. Textbook questions a. and b. of "Explore: Graph the Functions," p. 44**

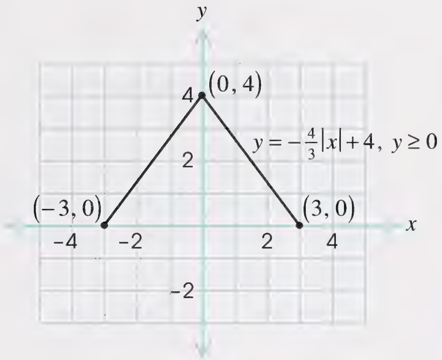
a. $y = -\frac{3}{4}|x| + 3,\ y \geq 0$

x	y
4	0
3	$\frac{3}{4}$
2	$1\frac{1}{2}$
1	$2\frac{1}{4}$
0	3
−1	$2\frac{1}{4}$
−2	$1\frac{1}{2}$
−3	$\frac{3}{4}$
−4	0



b. $y = -\frac{4}{3}|x| + 4,\ y \geq 0$

x	y
3	0
2	$1\frac{1}{3}$
1	$2\frac{2}{3}$
0	4
−1	$2\frac{2}{3}$
−2	$1\frac{1}{3}$
−3	$1\frac{1}{2}$



b. Textbook questions 1 to 7 of "Inquire," p. 44

1. Find the side lengths of triangle a. using the Pythagorean Theorem.

$$\begin{aligned}\text{side length} &= \sqrt{4^2 + 3^2}\\ &= \sqrt{16+9}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

The side lengths are 5, and the height is 3.

2. The slope of the left side of triangle a. is $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$.

The slope of the right side of triangle a. is $\frac{\text{rise}}{\text{run}} = \frac{-3}{4}$ or $-\frac{3}{4}$.

3. Find the side lengths of triangle b. using the Pythagorean Theorem.

$$\begin{aligned}\text{side length} &= \sqrt{3^2 + 4^2}\\ &= \sqrt{9+16}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

The side lengths are 5, and the height is 4.

Section 2: Activity 1 (continued)

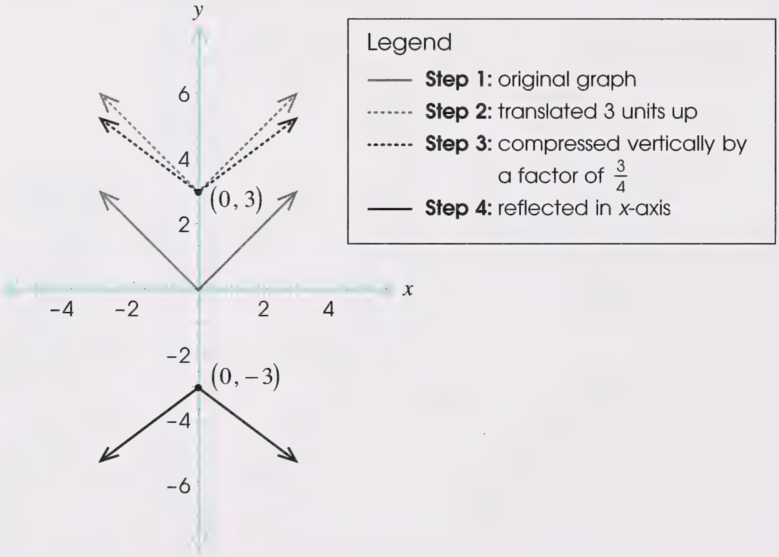
4. The slope of the left side of triangle b. is $\frac{\text{rise}}{\text{run}} = \frac{4}{3}$.

The slope of the right side of triangle b. is $\frac{\text{rise}}{\text{run}} = \frac{-4}{3}$ or $-\frac{4}{3}$.

5. The graph of $y = -\frac{3}{4}|x| + 3$ is the graph of $y = |x|$ compressed vertically by a factor of $\frac{3}{4}$, then reflected in the x-axis, and then translated 3 units up.

6. The graph of $y = -\frac{4}{3}|x| + 4$ is the graph of $y = |x|$ expanded vertically by a factor of $\frac{4}{3}$, then reflected in the x-axis, and then translated 4 units up.

7. In questions 5 and 6, the transformations must be applied in a particular order. Suppose, for example, in question 5 you first translated $y = |x|$ 3 units up, then compressed the graph vertically by a factor of $\frac{3}{4}$, and finally reflected the graph in the x-axis. You would obtain the following graph.



2. a. **Textbook questions 1, 5, 9, 14, 18, 19.a., 23, 31, 32, and 39 of "Practice," pp. 47 and 48**

1. The graph of $y=2f(x)+3$ is the graph of $y=f(x)$ expanded vertically by a factor of 2 and then translated 3 units up.

5. The graph of $y=f\left(\frac{1}{2}x\right)-6$ is the graph of $y=f(x)$ expanded horizontally by a factor of 2 and then translated 6 units down.

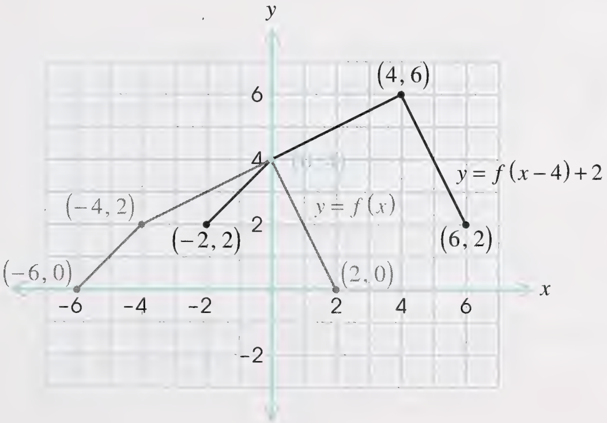
9. The graph of $y=f(-(x+1))-1$ is the graph of $y=f(x)$ reflected in the *y*-axis and then translated 1 unit to the left and 1 unit down.

14. The graph of $y=-2f(4(x-2))$ is the graph of $y=f(x)$ compressed horizontally by a factor of $\frac{1}{4}$, then expanded vertically by a factor of 2, then reflected in the *x*-axis, and then translated 2 units to the right.

18. Rewrite the function $y=f(3x-6)+8$ as $y=f(3(x-2))+8$.

The graph of $y=f(3(x-2))+8$ is the graph of $y=f(x)$ compressed horizontally by a factor of $\frac{1}{3}$ and then translated 2 units to the right and 8 units up.

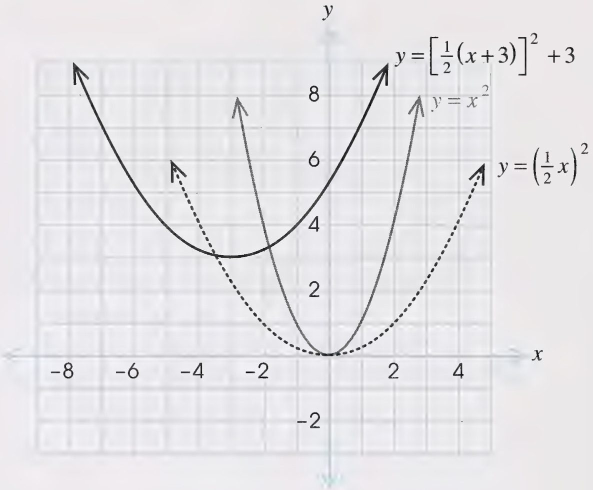
19. **a.** The graph of $y=f(x-4)+2$ is the graph of $y=f(x)$ translated 4 units to the right and 2 units up.



Section 2: Activity 1 (continued)

23. The graph of $y=\left(\frac{1}{2}x\right)^2$ is the graph of $y=x^2$ expanded horizontally by a factor of 2.

The graph of $y=\left[\frac{1}{2}(x+3)\right]^2+3$ is the graph of $y=\left(\frac{1}{2}x\right)^2$ translated 3 units to the left and 3 units up.



31. Rewrite $y=-\frac{1}{2}f(2x+6)-2$ as $y=-\frac{1}{2}f(2(x+3))-2$.

The graph of $y=-\frac{1}{2}f(2(x+3))-2$ is the graph of $y=f(x)$ compressed horizontally by a factor of $\frac{1}{2}$, then compressed vertically by a factor of $\frac{1}{2}$, then reflected in the x-axis, and then translated 3 units to the left and 2 units down.

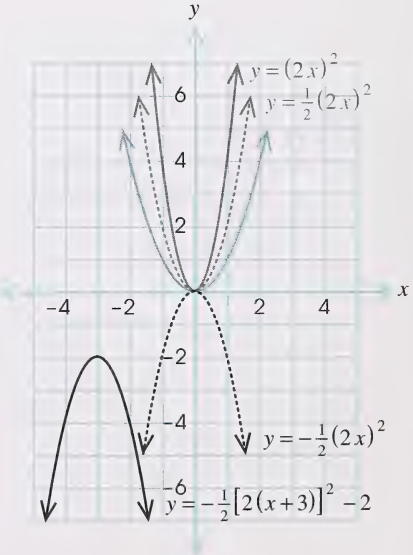
Step 1: Since $f(x)=x^2$, draw the graph of $y=x^2$.

Step 2: Draw the graph of $y=(2x)^2$. This graph is the graph of $y=x^2$ compressed horizontally by a factor of $\frac{1}{2}$.

Step 3: Draw the graph of $y=\frac{1}{2}(2x)^2$. This is the graph of $y=(2x)^2$ compressed vertically by a factor of $\frac{1}{2}$.

Step 4: Draw the graph of $y=-\frac{1}{2}(2x)^2$. This is the graph of $y=\frac{1}{2}(2x)^2$ reflected in the x-axis.

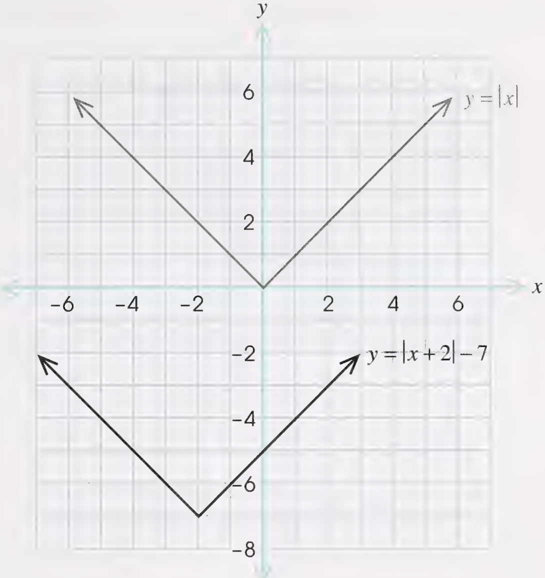
Step 5: Draw the graph of $y=-\frac{1}{2}[2(x+3)]^2-2$. This is the graph of $y=-\frac{1}{2}(2x)^2$ translated 3 units to the left and 2 units down.



32. The graph of $y = f(x+2) - 7$ is the graph of $y = f(x)$ translated 2 units to the left and 7 units down.

Because $f(x) = |x|$, you must draw the graph of $f(x) = |x+2| - 7$.

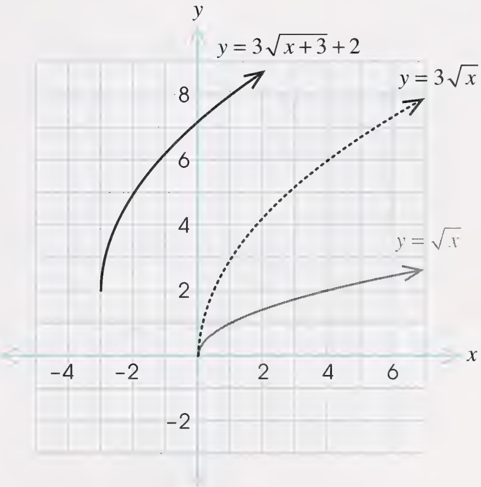
First, draw the graph of $y = |x|$; then translate the graph of $y = |x|$ 2 units to the left and 7 units down to obtain the graph of $y = |x+2| - 7$.



39. The graph of $y = 3f(x+3) + 2$ is the graph of $y = f(x)$ expanded vertically by a factor of 3 and then translated 3 units to the left and 2 units up.

Because $f(x) = \sqrt{x}$, you must draw the graph of $y = 3\sqrt{x+3} + 2$.

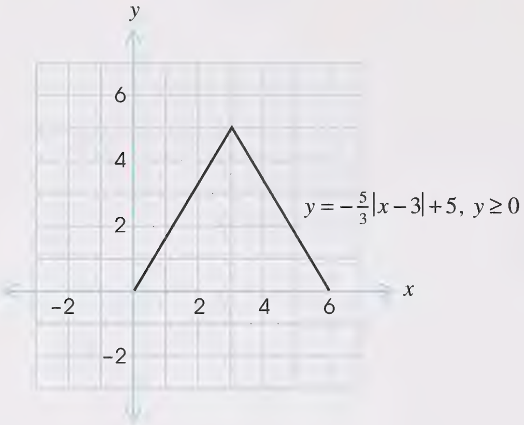
First, draw the graph of $y = \sqrt{x}$. Second, expand the graph of $y = \sqrt{x}$ vertically by a factor of 3 to obtain the graph of $y = 3\sqrt{x}$. Finally, translate the graph of $y = 3\sqrt{x}$ 3 units to the left and 2 units up to obtain the graph of $y = 3\sqrt{x+3} + 2$.



Section 2: Activity 1 (continued)

b. Textbook questions 49, 50, and 56 of "Applications and Problem Solving," pp. 48 and 49

49. a.



b. Use the Pythagorean Theorem to find the side lengths of the isosceles triangle.

$$\begin{aligned}\text{side length} &= \sqrt{3^2+5^2}\\ &= \sqrt{9+25}\\ &= \sqrt{34}\\ &\doteq 5.8\end{aligned}$$

The side lengths are approximately 5.8 units, and the height is 5 units.

c. The slope $\left(\frac{\text{rise}}{\text{run}}\right)$ of the roof on the left is $\frac{5}{3}$; and the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ of the roof on the right is $-\frac{5}{3}$.

d. The graph of $y=-\frac{5}{3}|x-3|+5$ is the graph of $y=|x|$ expanded vertically by a factor of $\frac{5}{3}$, then reflected in the x-axis, and then translated 3 units to the right and 5 units up.

The graph of $y=-\frac{5}{3}|x-3|+5$, where $y\geq 0$, is the portion of the graph of $y=-\frac{5}{3}|x-3|+5$ that lies on or above the x-axis.

50. **a.** The graph of $y=-5(x-4)^2+80$ is the graph of $y=x^2$ expanded vertically by a factor of 5, then reflected in the *x*-axis, and then translated 4 units to the right and 80 units up.

b. The minimum point on the graph of $y=x^2$ is $(0, 0)$. This point is invariant under a vertical expansion. However, when the graph of $y=x^2$ is reflected in the *x*-axis, $(0, 0)$ becomes the maximum point on the graph. This point is then translated 4 units to the right and 80 units up to $(4, 80)$. Therefore, the flare reaches a maximum height of 80 m after 4 s.

56. The inverse of $y=x^3$ is $x=y^3$ or $y=\sqrt[3]{x}$. Therefore, the graph of $y=\sqrt[3]{x+1}-2$ is the graph of $y=x^3$ reflected in the line $y=x$ and then translated 1 unit to the left and 2 units down.

3. **a.** **Textbook questions 44, 45, and 47 of "Practice," p. 48**

44. When $y=(x+2)^2+3$ is reflected in the *x*-axis, the equation becomes

$$\begin{aligned}y&=-\left[(x+2)^2+3\right]\\&=-(x+2)^2-3\end{aligned}$$

When $y=-(x+2)^2-3$ is reflected in the *y*-axis, the equation becomes

$$\begin{aligned}y&=-\left[(-x)+2\right]^2-3\\&=-\left[-1(x-2)\right]^2-3\\&=-(x-2)^2-3\end{aligned}$$

The equation of the image is $y=-(x-2)^2-3$.

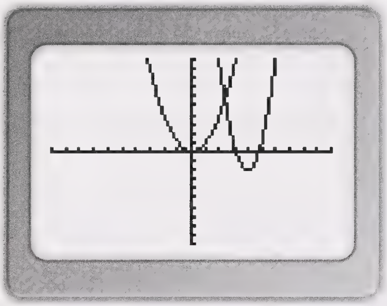
Section 2: Activity 1 (continued)

45. When $f(x)=x^2$ is expanded vertically by a factor of 3, it becomes $f(x)=3x^2$.

When $f(x)=3x^2$ is translated 4 units to the right and 2 units down, it becomes $f(x)=3(x-4)^2-2$.

Therefore, $k(x)=3(x-4)^2-2$.

Check your answer using a graphing calculator.



Note that the vertex is at $(4,-2)$.

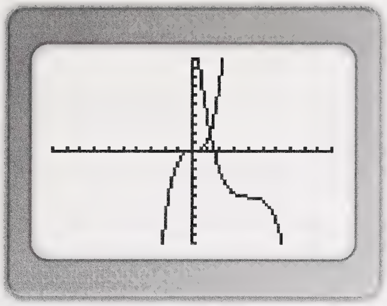
47. When $f(x)=x^3$ is compressed vertically by a factor of $\frac{1}{3}$, it becomes $f(x)=\frac{1}{3}x^3$.

When $f(x)=\frac{1}{3}x^3$ is reflected in the x-axis, it becomes $f(x)=-\frac{1}{3}x^3$.

When $f(x)=-\frac{1}{3}x^3$ is translated 4 units to the right and 5 units down, it becomes $f(x)=-\frac{1}{3}(x-4)^3-5$.

Therefore, $g(x)=-\frac{1}{3}(x-4)^3-5$.

Check your answer using a graphing calculator.



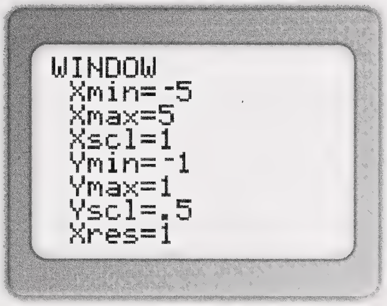
b. Textbook questions 52.a. and 54 of "Applications and Problem Solving," pp. 48 and 49

52. a. When $f(x)=\frac{x}{x^2+1}$ is reflected in the x-axis, it becomes $f(x)=-\frac{x}{x^2+1}$.

When $f(x)=-\frac{x}{x^2+1}$ is translated 3 units to the left, it becomes $f(x)=-\frac{x+3}{(x+3)^2+1}$.

Therefore, the resulting equation is $f(x)=-\frac{x+3}{(x+3)^2+1}$.

Check your answer using a graphing calculator.



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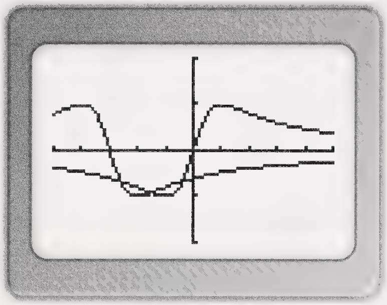
WINDOW
 Xmin=⁻5
 Xmax=5
 Xscl=1
 Ymin=⁻1
 Ymax=1
 Yscl=.5
 Xres=1
```



54. a. When $y=f(x)$ is expanded vertically by a factor of 3, it becomes $y=3f(x)$.

When $y=3f(x)$ is reflected in the y-axis, it becomes $y=3f(-x)$.

Compare $y=3f(-x)$ with $y=af(k(x-h))+q$. Therefore, $a=3$, $k=-1$, $h=0$, and $q=0$.

Section 2: Activity 1 (continued)

b. When $y=f(x)$ is compressed vertically by a factor of $\frac{1}{3}$, it becomes $y=\frac{1}{3}f(x)$.

When $y=\frac{1}{3}f(x)$ is expanded horizontally by a factor of 2, it becomes $y=\frac{1}{3}f\left(\frac{1}{2}x\right)$.

When $y=\frac{1}{3}f\left(\frac{1}{2}x\right)$ is translated 6 units to the right and 1 unit down, it becomes $y=\frac{1}{3}f\left(\frac{1}{2}(x-6)\right)-1$.

Compare $y=\frac{1}{3}f\left(\frac{1}{2}(x-6)\right)-1$ with $y=af\left(k(x-h)\right)+q$. Therefore, $a=\frac{1}{3}$, $k=\frac{1}{2}$, $h=6$, and $q=-1$.

c. When $y=f(x)$ is expanded vertically by a factor of 2, it becomes $y=2f(x)$.

When $y=2f(x)$ is compressed horizontally by a factor of $\frac{1}{2}$, it becomes $y=2f(2x)$.

When $y=2f(2x)$ is reflected in the *x*-axis, it becomes $y=-2f(2x)$.

When $y=-2f(2x)$ is reflected in the *y*-axis, it becomes $y=-2f(-2x)$.

When $y=-2f(-2x)$ is translated 7 units to the left and 4 units up, it becomes $y=-2f\left(-2(x+7)\right)+4$.

Compare $y=-2f\left(-2(x+7)\right)+4$ with $y=af\left(k(x-h)\right)+q$. Therefore, $a=-2$, $k=-2$, $h=-7$, and $q=4$.

4. Textbook questions 1 to 6 of Investigation 1, "Ages of Cats and Dogs," p. 60

1.
$$\begin{aligned} h &= 4a+15 \\ &= 4(8)+15 \\ &= 47 \end{aligned}$$

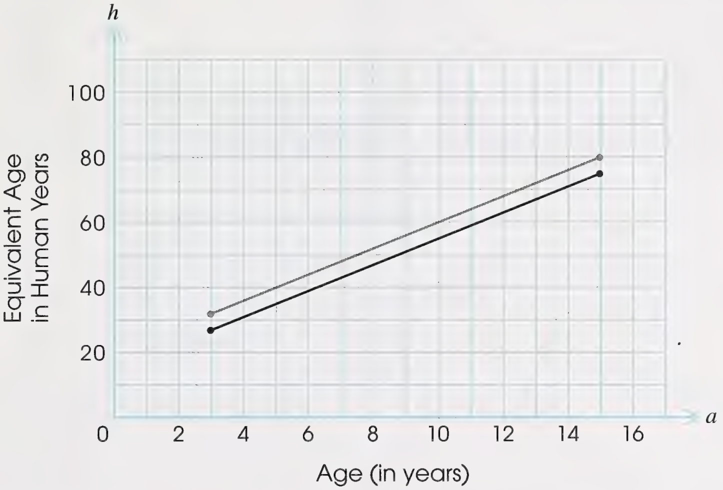
The age of an 8-year-old cat is equivalent to 47 human years.

2.

$$\begin{aligned} h &= 4a+20 \\ 64 &= 4a+20 \\ 44 &= 4a \\ 4a &= 44 \\ a &= 11 \end{aligned}$$

The dog is 11 years old.

3. **a.**



b. The graph of $h = 4a + 15$ is the graph of $h = 4a + 20$ translated 5 units down.

4. If a small dog and a cat were born on the same day and are over 3 years old, then the dog is 5 years older in human years than the cat.

$$\begin{aligned} h_{\text{dog}} &= 4a+20 \\ &= (4a+15)+5 \\ &= h_{\text{cat}}+5 \end{aligned}$$

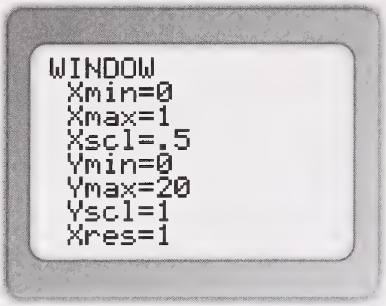
5. The slope of the graphs represents the rate at which they age in human years. Because the graphs are parallel after three years, the slopes of the graphs are equal. Therefore, cats and small dogs age at the same rate after the age of 3.

6. Cats and dogs do not age at the same rates from birth. A cat ages 27 human years in its first 3 years, whereas a small dog ages 32 human years in its first 3 years.

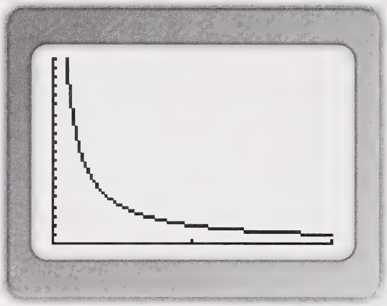
Section 2: Activity 2

1. a. Textbook questions a. to d. of “Explore: Graph the Function,” p. 50

a.



```
WINDOW
 Xmin=0
 Xmax=1
 Xscl=.5
 Ymin=0
 Ymax=20
 Yscl=1
 Xres=1
```



b. The graph is in the first quadrant because the density, D, and the specific volume, v, are positive.

c.

$$\begin{aligned} D &= \frac{1}{v} \\ 10.5 &= \frac{1}{v} \\ 10.5v &= 1 \\ v &= \frac{1}{10.5} \\ &\doteq 0.0952 \end{aligned}$$

The specific volume of silver is about 0.0952 cm^3/g.

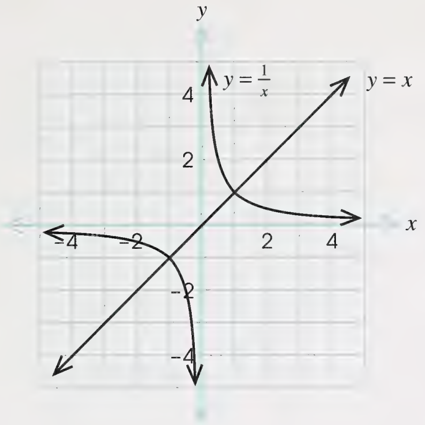
d.

$$\begin{aligned} D &= \frac{1}{v} \\ &= \frac{1}{0.285} \\ &\doteq 3.5 \end{aligned}$$

The density of diamond is about 3.5 g/cm^3.

b. Textbook questions 1 to 7 of "Inquire," pp. 50 and 51

1. a.



b. The graph of $y = \frac{1}{x}$ lies in the first and third quadrants because both x and y can be negative. This differs from the graph of $D = \frac{1}{v}$ because D and v are non-negative, restricting the graph to the first quadrant.

2. Refer to the answer to question 1.a.

3. When $x = 0$, $\frac{1}{x}$ is undefined because division by 0 is undefined.

4. There are two points on the graph where x and $\frac{1}{x}$ are equal. These are the points $(1, 1)$ and $(-1, -1)$ on $y = \frac{1}{x}$.

5. a. When x is positive, $\frac{1}{x}$ is positive.

b. When x is negative, $\frac{1}{x}$ is negative.

6. a. As x increases, $\frac{1}{x}$ decreases.

b. As x decreases, $\frac{1}{x}$ increases.

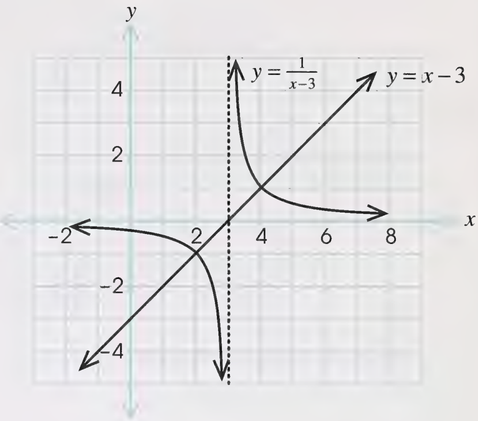
7. a. As $|x|$ approaches 0, $\left|\frac{1}{x}\right|$ approaches infinity.

b. As $|x|$ increases, $\left|\frac{1}{x}\right|$ approaches zero.

Section 2: Activity 2 (continued)

2. a. Textbook questions 3, 7, 10, 13, 17, and 19 of "Practice," p. 56

3. Draw the graph of $f(x)=x-3$ or $y=x-3$. The function has a zero at $x=3$ and its graph has a y-intercept of -3.



The function $y=\frac{1}{x-3}$ is undefined when $x-3=0$, that is, when $x=3$. Draw a vertical asymptote at $x=3$.

The graph of $y=\frac{1}{x-3}$ is positive when $x>3$ because $y=x-3$ is positive for these values.

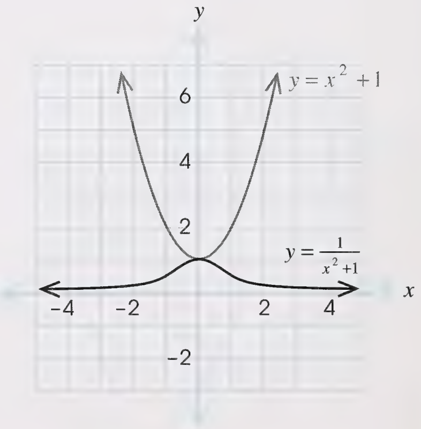
The graph of $y=\frac{1}{x-3}$ is negative when $x<3$ because $y=x-3$ is negative for these values.

As x approaches 3, $|x-3|$ approaches 0; so, $\frac{1}{|x-3|}$ approaches infinity.

As $|x|$ gets large, $|x-3|$ also gets large; so, $\frac{1}{|x-3|}$ approaches 0.

The line $y=0$ is a horizontal asymptote of $y=\frac{1}{x-3}$.

7. The graph of $f(x)=x^2+1$ or $y=x^2+1$ is a parabola that opens upward from its vertex $(0,1)$.



Note that since $y=x^2+1$ does not intersect the x-axis, this function has no real zeros. So, $\frac{1}{x^2+1}$ is defined for all real numbers. Therefore, the graph of $y=\frac{1}{f(x)}$ or $y=\frac{1}{x^2+1}$ has no vertical asymptotes.

Because $f(x)$ is always positive, $\frac{1}{f(x)}$ is always positive.

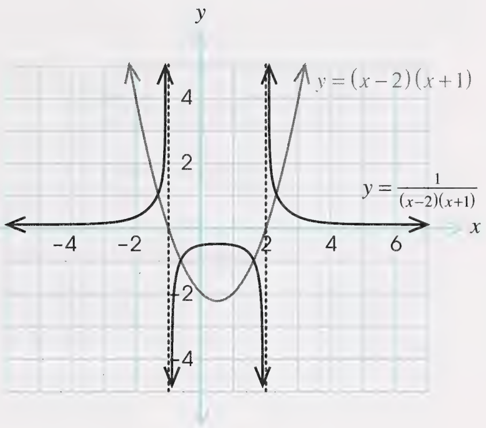
When $x=0$, $f(x)=1$ and $\frac{1}{f(x)}=1$.

As $|x|$ gets large, $f(x)$ gets large and $\frac{1}{f(x)}$ approaches 0.

The line $y=0$ is a horizontal asymptote of $y=\frac{1}{x^2+2}$.

10. Draw the graph of $f(x)=(x-2)(x+1)$ or $y=(x-2)(x+1)$. The x-intercepts are -1 and 2. The y-intercept is

$$\begin{aligned} f(0) &= (x-2)(0+1) \\ &= -2 \end{aligned}$$



The graph of $y=(x-2)(x+1)$ is a parabola that opens upward and has a vertex at $x=\frac{1}{2}$ (halfway between the zeros).

$$\begin{aligned} \therefore f\left(\tfrac{1}{2}\right) &= \left(\tfrac{1}{2}-2\right)\left(\tfrac{1}{2}+1\right) \\ &= \left(-\tfrac{3}{2}\right)\left(\tfrac{3}{2}\right) \\ &= -\tfrac{9}{4} \end{aligned}$$

The vertex of $y=f(x)$ is $\left(\frac{1}{2},-\frac{9}{4}\right)$. The corresponding point on $y=\frac{1}{f(x)}$ is $\left(\frac{1}{2},-\frac{4}{9}\right)$. The graph of $\frac{1}{f(x)}=\frac{1}{(x-2)(x+1)}$ is undefined at $x=2$ and $x=-1$. Draw vertical asymptotes along these lines.

Now, $f(x)=(x-2)(x+1)$ is positive when $x<-1$ or $x>2$. Therefore, $\frac{1}{f(x)}=\frac{1}{(x-2)(x+1)}$ is also positive for these values.

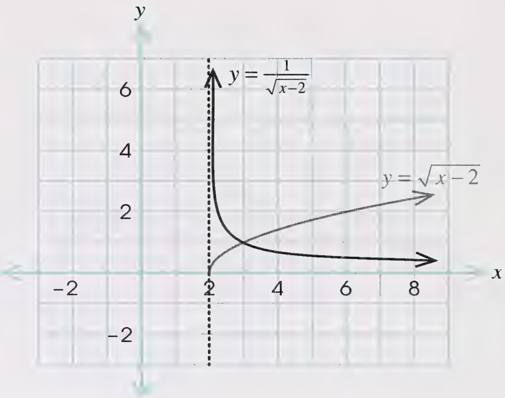
Also, $f(x)=(x-2)(x+1)$ is negative when $-1<x<2$. Therefore, $\frac{1}{f(x)}=\frac{1}{(x-2)(x+1)}$ is also negative for these values.

As x approaches -1 or 2, $\frac{1}{|f(x)|}$ approaches infinity. As $|x|$ becomes large, $|x-2|$ and $|x+1|$ become large and $\frac{1}{|f(x)|}$ approaches 0.

The line $y=0$ is a horizontal asymptote.

Section 2: Activity 2 (continued)

13. First, sketch the graph of $f(x)=\sqrt{x-2}$ or $y=\sqrt{x-2}$. This function is defined when $x-2\geq 0$ or when $x\geq 2$.

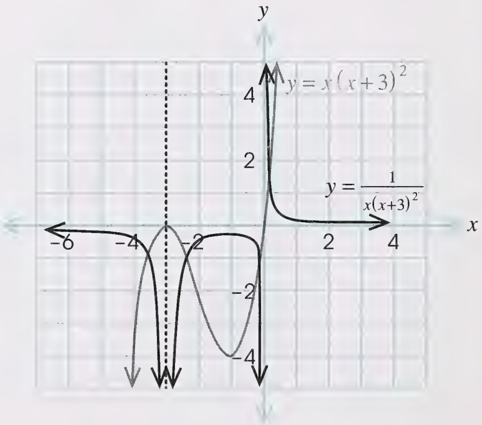


When $x=2$, $f(x)=0$. When $x>2$, $f(x)>0$. Therefore, the function $y=\frac{1}{f(x)}$ is defined only for $x>2$; and for these values of *x*, $\frac{1}{f(x)}>0$.

When *x* approaches 2 from the right, $f(x)$ approaches 0 and $\frac{1}{f(x)}$ approaches infinity. Therefore, the vertical line $x=2$ is an asymptote of the graph of $y=\frac{1}{f(x)}$ or $y=\frac{1}{\sqrt{x-2}}$.

As *x* becomes large, $f(x)$ becomes large and $\frac{1}{f(x)}$ approaches 0. The line $y=0$ is a horizontal asymptote of $y=\frac{1}{f(x)}$.

17. First, graph $f(x)=x(x+3)^2$ or $y=x(x+3)^2$.



The zeros of this function are $x=0$ and $x=-3$. When $x<0$ and $x\neq -3$, $f(x)<0$. When $x>0$, $f(x)>0$. Therefore, the function $y=\frac{1}{f(x)}$ or $y=\frac{1}{x(x+3)^2}$ is undefined when $x=0$ and $x=-3$. The vertical lines at $x=0$ and $x=-3$ are the vertical asymptotes of the graph of $y=\frac{1}{f(x)}$.

Also, when $x<0$ and $x\neq -3$, $\frac{1}{f(x)}$ is negative; when $x>0$, $\frac{1}{f(x)}$ is positive.

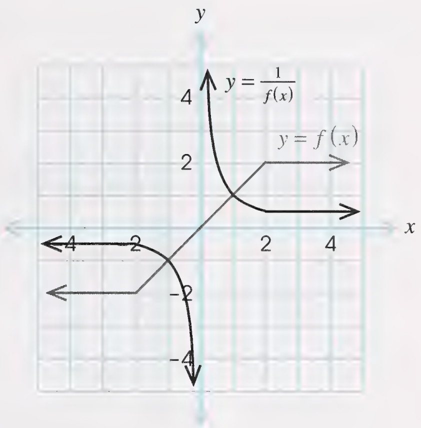
As *x* approaches 0 or -3, $|f(x)|$ approaches 0 and $\left|\frac{1}{f(x)}\right|$ approaches infinity. Therefore, the vertical lines $x=0$ and $x=-3$ are asymptotes of the graph of $y=\frac{1}{f(x)}$.

As $|x|$ becomes large, $|f(x)|$ becomes large and $\left|\frac{1}{f(x)}\right|$ approaches 0. The line $y=0$ is a horizontal asymptote of $y=\frac{1}{f(x)}$.

19. For $x \geq 2$, $f(x) = 2$ and $\frac{1}{f(x)} = \frac{1}{2}$.

For $x \leq -2$, $f(x) = -2$ and $\frac{1}{f(x)} = -\frac{1}{2}$.

At $x = 0$, $f(x) = 0$ and $\frac{1}{f(x)}$ is undefined. The vertical line $x = 0$ is an asymptote of $y = \frac{1}{f(x)}$. As $|x|$ approaches 0, $\left|\frac{1}{f(x)}\right|$ becomes very large.



b. Textbook questions 44 and 54 of "Applications and Problem Solving," p. 57

44. When $y = x^2$ is compressed horizontally by a factor of $\frac{1}{2}$, it becomes $y = (2x)^2$.

When $y = (2x)^2$ is shifted 4 units to the right and 3 units down, it becomes

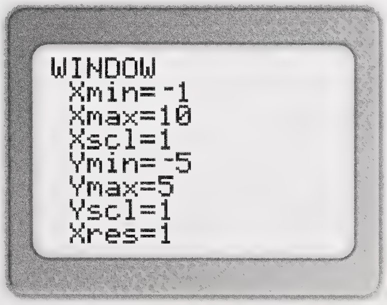
$$\begin{aligned} y &= [2(x-4)]^2 - 3 \\ &= 4(x-4)^2 - 3 \end{aligned}$$

When you take the reciprocal of $y = 4(x-4)^2 - 3$, it becomes

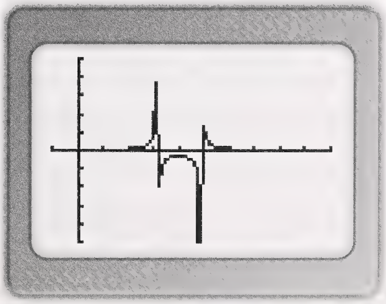
$$y = \frac{1}{4(x-4)^2 - 3}$$

Therefore, the equation is $y = \frac{1}{4(x-4)^2 - 3}$.

Check your answer using a graphing calculator.

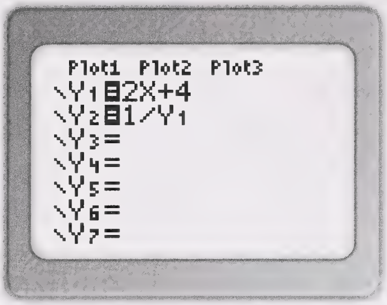


```
WINDOW
 Xmin=-1
 Xmax=10
 Xscl=1
 Ymin=-5
 Ymax=5
 Yscl=1
 Xres=1
```

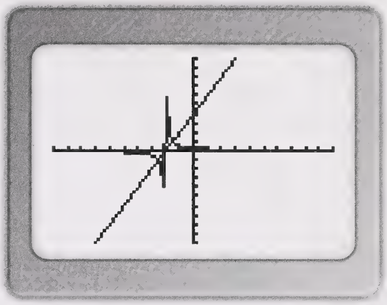


Section 2: Activity 2 (continued)

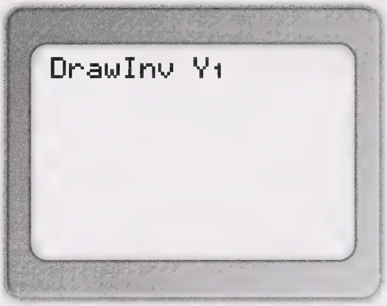
54. For the function $f(x)=2x+4$, $\frac{1}{f(x)}$ is the reciprocal function and $f^{-1}(x)$ is the inverse function.



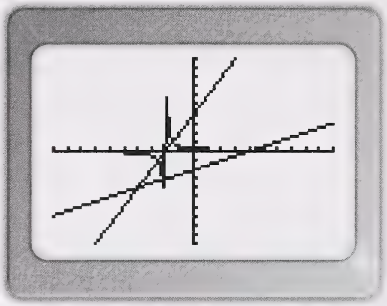
```
Plot1 Plot2 Plot3
\Y1=2X+4
\Y2=1/Y1
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



The graph of $y=\frac{1}{f(x)}$ is a hyperbola.



```
DrawInv Y1
```



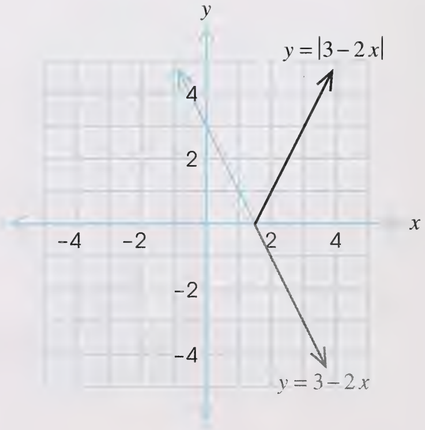
The graph of $f^{-1}(x)=\frac{1}{2}x-2$ is a line.

3. a. Textbook questions 28, 32, 34, and 41 of "Practice," pp. 56 and 57

28. Graph $f(x)=3-2x$ or $y=3-2x$.

The x-intercept of this graph is $\frac{3}{2}$.

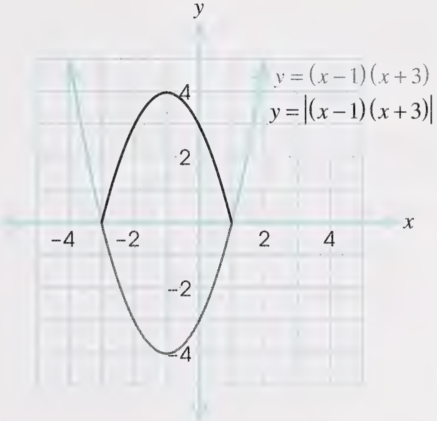
When $x>\frac{3}{2}$, $f(x)<0$. For these values of x, the graph of $y=|f(x)|$ is a reflection of the graph of $y=f(x)$ in the x-axis. For all other values of x, the graphs of $y=f(x)$ and $y=|f(x)|$ are identical (shown in colour).



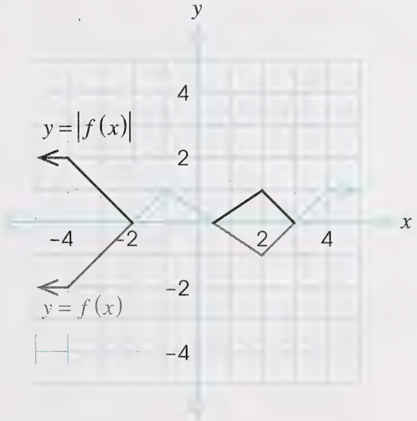
32. Graph $f(x)=(x-1)(x+3)$ or $y=(x-1)(x+3)$.

The x-intercepts of this graph are -3 and 1.

When $-3<x<1$, $f(x)<0$. For these values of x, the graph of $y=|f(x)|$ is a reflection of the graph of $y=f(x)$ in the x-axis. For all other values of x, the graphs of $y=f(x)$ and $y=|f(x)|$ are identical (shown in colour).



34. Reflect in the x-axis the portion of $y=f(x)$ that lies below the x-axis. All other portions of $y=f(x)$ and $y=|f(x)|$ are identical (shown in colour).



Section 2: Activity 2 (continued)

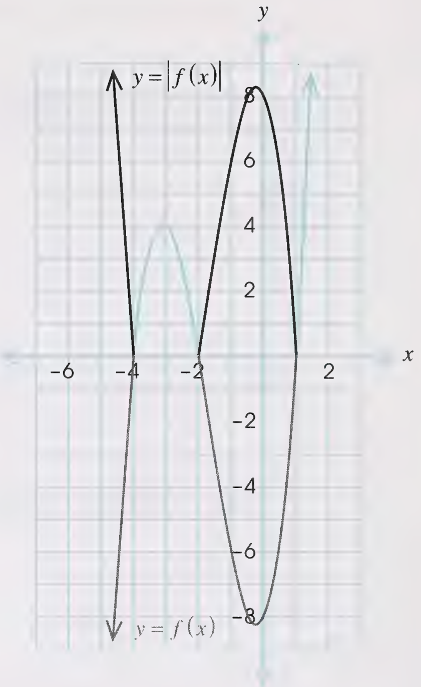
41. First, graph $y=(x-1)(x+2)(x+4)$.

The x-intercepts are -4, -2, and 1.

To find the y-intercept, let $x=0$.

$$\begin{aligned}\therefore\ y&=(x-1)(x+2)(x+4)\\&=(0-1)(0+2)(0+4)\\&=-8\end{aligned}$$

Use these intercepts to draw the curve. When $x<-4$ and $-2<x<1$, $f(x)<0$. To draw $y=|f(x)|$, reflect these portions of $y=f(x)$ in the x-axis. All other portions of $y=f(x)$ and $y=|f(x)|$ are identical (shown in colour).



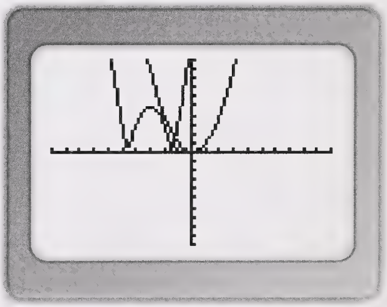
b. Textbook questions 45, 47, 50, and 52 of "Applications and Problem Solving," p. 57

45. When $y=x^2$ is expanded vertically by a factor of 2, it becomes $y=2x^2$.

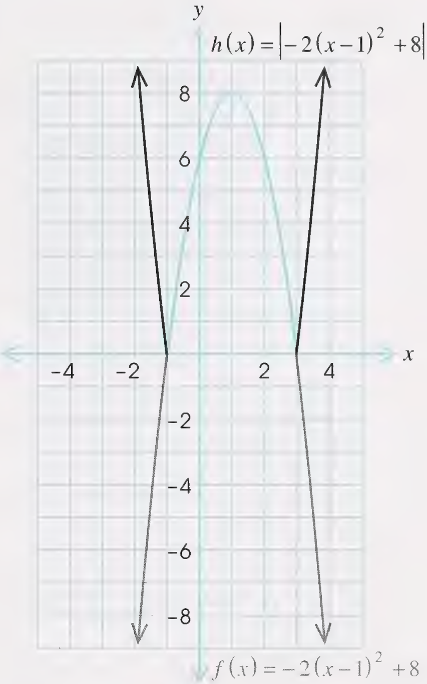
When $y=2x^2$ is translated 3 units to the left and 5 units down, it becomes $y=2(x+3)^2-5$.

When you take the absolute value of $y=2(x+3)^2-5$, it becomes $y=|2(x+3)^2-5|$.

Therefore, the equation is $y=|2(x+3)^2-5|$.



47. First, sketch $f(x) = -2(x-1)^2 + 8$ or $y = -2(x-1)^2 + 8$. Its graph is a parabola that opens downward from its vertex $(1, 8)$.



Because the absolute value function transforms the portions of the graph that lie below the *x*-axis, you must first determine the *x*-intercepts.

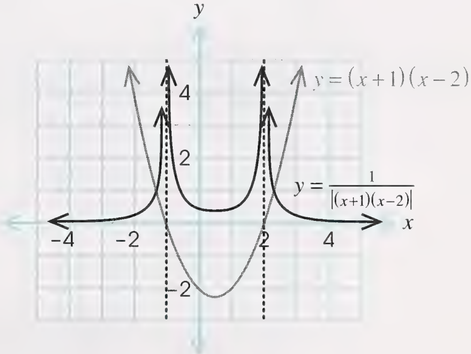
To determine the *x*-intercepts, let $y = 0$.

$$\therefore\ y = -2(x-1)^2 + 8$$
$$0 = -2(x-1)^2 + 8$$
$$2(x-1)^2 = 8$$
$$(x-1)^2 = 4$$
$$x - 1 = \pm 2$$

$$\therefore\ x - 1 = 2 \quad \text{or} \quad x - 1 = -2$$
$$x = 3 \qquad\qquad x = -1$$

When $x < -1$ or $x > 3$, $f(x) < 0$. To draw $h(x) = |-2(x-1)^2 + 8|$, reflect these portions of $y = f(x)$ in the *x*-axis. All other portions of $y = f(x)$ and $y = h(x)$ are identical (shown in colour).

50. First, draw $y = (x+1)(x-2)$. Its graph is a parabola that crosses the *x*-axis at -1 and 2 and the *y*-axis at -2.



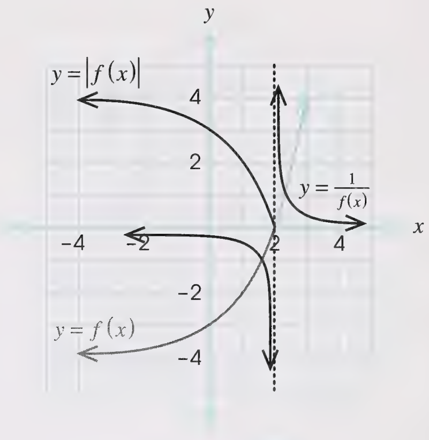
The graph of $f(x) = \frac{1}{|(x+1)(x-2)|}$ will be undefined at $x = -1$ and $x = 2$; therefore, it has vertical asymptotes along these lines.

When $|x|$ becomes large, $f(x)$ becomes small and approaches 0; therefore, the line $y = 0$ is a horizontal asymptote.

Section 2: Activity 2 (continued)

52. First, draw $f(x)=2^{x}-4$.

x	y
-3	-3.875
-2	-3.75
-1	-3.5
0	-3
1	-2
2	0
3	4



Note: The colour portion shows where the graphs of $y=f(x)$ and $y=|f(x)|$ overlap.

Next, graph $y=\frac{1}{f(x)}$.

Because 2 is the zero of $y=f(x)$, the line $x=2$ is a vertical asymptote for $y=\frac{1}{f(x)}$.

For $x<2$, $f(x)<0$ and $\frac{1}{f(x)}<0$.

For $x>2$, $f(x)>0$ and $\frac{1}{f(x)}>0$.

As x approaches negative infinity, $f(x)$ approaches -4 and $\frac{1}{f(x)}$ approaches $-\frac{1}{4}$. As x approaches positive infinity, $f(x)$ approaches positive infinity and $\frac{1}{f(x)}$ approaches 0.

Lastly, graph $y=|f(x)|$ by reflecting in the x-axis the portion of $y=f(x)$ that lies below the x-axis. All other portions of $y=f(x)$ and $y=|f(x)|$ are identical (shown in colour).

4. Textbook questions 1 to 12 of "Mental Math," p. 3

1. $(-2, 5+3)=(-2, 8)$

2. $(-2, 5-6)=(-2, -1)$

3. $(-2-3, 5)=(-5, 5)$

4. $(-2+4, 5)=(2, 5)$

5. $(-2+1, 5-2)=(-1, 3)$

6. $(-2-5, 5+2)=(-7, 7)$

7. 2 units to the right

8. 4 units down

9. 6 units to the left

10. 5 units up

11. 4 units to the right and 1 unit up

12. 3 units to the left and 2 units down

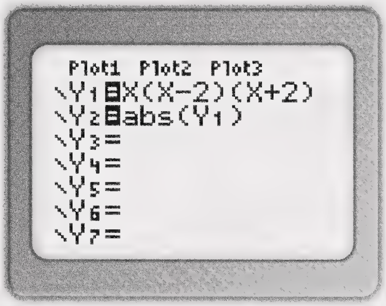
Section 2: Follow-up Activities

Extra Help

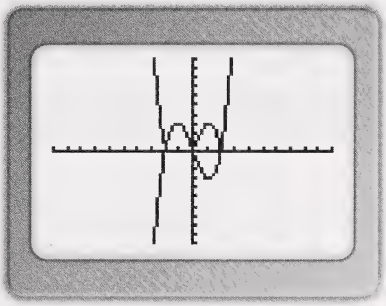
1. Enter $y=x(x-2)(x+2)$ as function Y_1 by pressing the following:

Y= X,T,Θ,n (X,T,Θ,n − 2) (X,T,Θ,n + 2) ENTER

Press GRAPH to obtain the graphs Y_1 and Y_2.



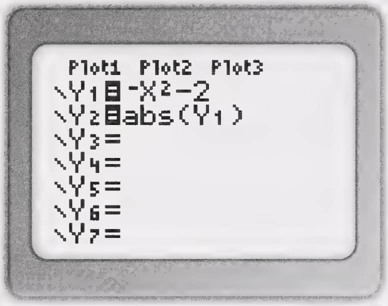
```
Plot1 Plot2 Plot3
\Y1=X(X-2)(X+2)
\Y2=abs(Y1)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



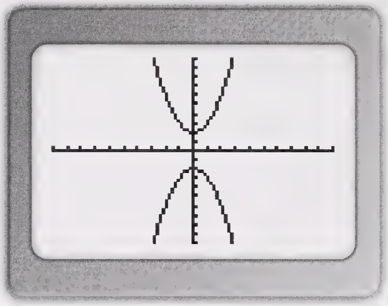
Extra Help (continued)

2. Clear Y_1, and enter the new function; then graph Y_1 and Y_2.

Y= (−) X,T,Θ,n x^2 − 2 GRAPH

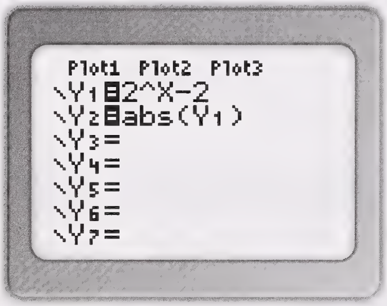


```
Plot1 Plot2 Plot3
\Y1=-X²-2
\Y2=abs(Y1)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

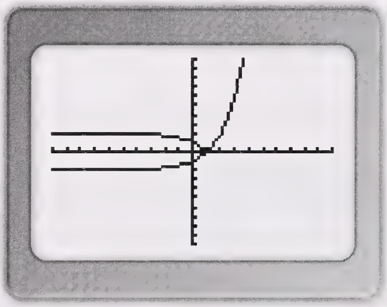


3. Clear Y_1, and enter the new function; then graph Y_1 and Y_2.

Y= 2 ^ X,T,Θ,n − 2 GRAPH



```
Plot1 Plot2 Plot3
\Y1=2^X-2
\Y2=abs(Y1)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



Enrichment

1. a. Textbook questions 1 to 4 of Investigation 1, "Using Transformations," pp. 61 and 62

1. The frieze pattern is created by repeatedly sliding or translating the basic design 7 units to the right.

2. The frieze pattern is created by reflecting the basic design across a vertical line lying one-half of a unit to its right. This procedure repeated until the entire pattern is formed.

3. The upper row of the frieze pattern is created by repeatedly translating the basic design 7 units to the right. Once the upper row is formed, the entire row is reflected across a horizontal line lying one-half of a unit below the upper row.

4. The frieze pattern is obtained by first reflecting the basic pattern across a vertical line lying one-half of a unit to its right. Then the image is rotated a half turn. This process is repeated until the entire frieze pattern is formed.

b. Textbook questions 1 to 4 of Investigation 2, "Transforming Triangles," pp. 62 and 63

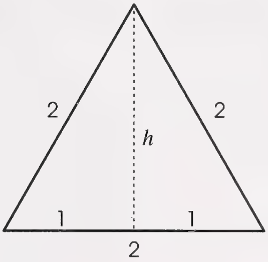
1. $h^2 + 1^2 = 2^2$

$h^2 + 1 = 4$

$h^2 = 3$

$h = \sqrt{3}$

The exact height of each triangle is $\sqrt{3}$ units.



2. a. $$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}}\\ &= \frac{\sqrt{3}}{1}\\ &= \sqrt{3}\end{aligned}$$

The slope of segment 1 is $\sqrt{3}$.

b. Use the slope and y-intercept formula.

$$\begin{aligned}y &= mx + b\\ &= \sqrt{3}x + 0\\ &= \sqrt{3}x\end{aligned}$$

$m = \sqrt{3}$ and $b = 0$

An equation for segment 1 is $y = \sqrt{3}x$.

Enrichment (continued)

3. **a.** Segment 2 is the reflection of segment 1 in the *y*-axis. Therefore, substitute $-x$ for x.

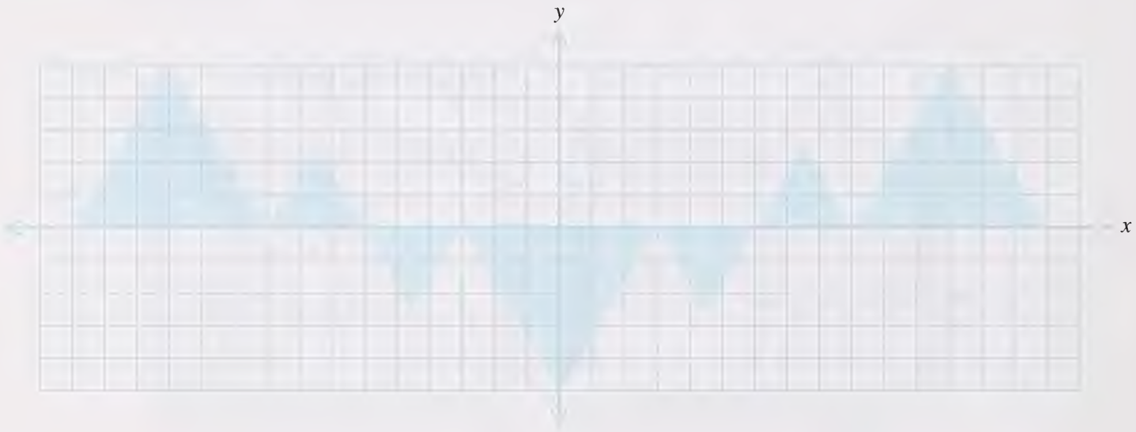
$$\begin{aligned} y &= \sqrt{3}(-x) \\ &= -\sqrt{3}x \end{aligned}$$

An equation for segment 2 is $y = -\sqrt{3}x$.

b. Segment 3 is segment 2 translated 2 units to the right. Therefore, an equation for segment 3 is $y = -\sqrt{3}(x-2)$.

c. Segment 4 is segment 1 translated 2 units to the left. Therefore, an equation for segment 4 is $y = \sqrt{3}(x+2)$.

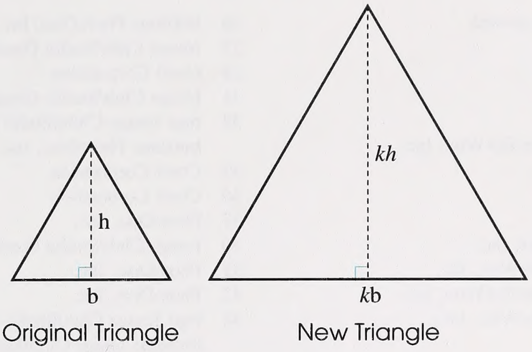
4. Answers may vary. A sample answer is given.



Problem

If the base and altitude of an equilateral triangle are stretched by a factor of *k*, how does the area of the new triangle compare with the area of the original triangle?

Solution



$$A_{\text{original triangle}} = \frac{1}{2}bh$$

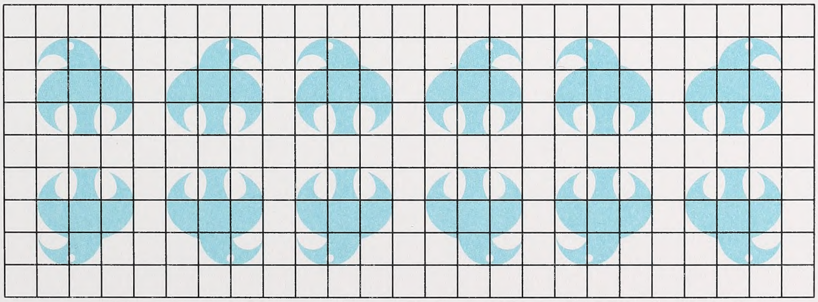
$$A_{\text{new triangle}} = \frac{1}{2}(kb)(kh)$$
$$= \frac{1}{2}bhk^2$$

$$\therefore \frac{A_{\text{new triangle}}}{A_{\text{original triangle}}} = \frac{\frac{1}{2}bhk^2}{\frac{1}{2}bh}$$
$$= k^2$$

The area of the new triangle is k^2 times larger than the original triangle.

2. Answers may vary. Frieze patterns may appear on carpets, furniture, wallpaper, woven baskets, and so on.

3. Answers may vary. A sample frieze pattern is given.



Credits

All clip art graphics are commercially owned.

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Pure Mathematics 30
Student Module Booklet
Module 1

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